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NEW PHYSICAL CONSTANTS FROM DIMENSIONAL ANALYSIS

BY

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ABSTRACT

A specific form of dimensional analysis has been employed to show that the electromagnetic gauss unit (for intensity of magnetization) is equivalent to a simple cgs quantity of 1.759×10^{7} sec^{-1}. Then, considering equivalences for known intensities of magnetization of the earth and certain stars in the hypothesis of P. M. S. Blackett (concerning the magnetic fields of massive rotating bodies), it is concluded from the dimensional method that $\mu_b = 1.295 \times 10^{-4}$cm g^{-1} is a valid constant of astronomy and classical mechanics. Further, it has been noted that the two quantities $G\mu_b^{-2}$ and $m_e^3c^3\hbar^{-1}\alpha_e^{-2}$ were each equal to the same quantity, $F_g = 3.978 \times 10^{8}$g cm sec^{-2}, signifying phenomenologic interrelationships among electromagnetic, quantum, and classical physics. From this it has then been concluded that another quantity $M_s = m_e\alpha_e^{-1} = 1.248 \times 10^{-25}$ g may represent a general constant of all submicroscopic physics.

Thus, the three quantities μ_b, M_s, and F_g are suggested as new physical constants. As formulated, the latter quantity F_g appears to correlate the mechanical and magnetic phenomena of either astronomic or submicroscopic particles and media; therefore, it is proposed as a broadly applicable constant of nature.

INTRODUCTION

Magnetism occupies an important place in modern science and philosophy. It represents a stubborn enigma of particulate physics, though it has long been recognized as a quite specific and measurable property of spinning bodies, such as electrons, atomic nuclei, the earth, the sun, and the stars. As an apparent "dimension" of the total universe, and its particles or fields of action-reaction, it has escaped the comprehension of those who would discover its source or penetrate its physical meanings in natural order. Strangely, success in the technology of magnetism has far outstripped that enjoyed in its science and theory. It appears that its subjective aura is destined to shroud the

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fundamental understanding of matter and energy, at least in the deterministic or mechanical sense.

The purpose of this study was to reveal by novel dimensional methods any simple interrelationships among constant physical quantities which might partially illuminate the mystery of magnetism as a phenomenon of particles or fields. If such illumination were attainable, it was hopefully conceived that many related mysteries of both astro- and submicrophysics would become vulnerable to a precise attack by theoretical and experimental investigation.

The need for such a result may not be immediately obvious from the modern viewpoint of strictest indeterminism; however, it was felt that lack of classical mechanisms or models for interpretations of magnetic phenomena may have seriously restricted the "vision" of science theory and philosophy. It will be recognized that science has not yet produced widely-accepted or lucid answers to the physical meanings of phenomena such as (1) electronic charge, fine structure, or magnetic moment; (2) nuclear magnetic moments or spins; or (3) terrestrial and stellar magnetism. If such meanings were clearly and quantitatively revealed, it is conceivable that major uncertainties of physical theory might be perspicuously resolved, e.g. the nature of nuclear binding forces, "new particle" phenomena, or terrestrial magnetic fluctuations.

The scope of the study has included a novel approach to interpretations of certain controversial subjects, for example, (*a*) the equations and units of measurement in electricity and magnetism (**1, 2**),[2] (*b*) P. M. S. Blackett's hypothesis (**3, 4**) regarding the magnetic field of massive rotating bodies, and (*c*) interrelationships among physical constants, laws, and hypotheses (**5, 6**).

CGS EQUIVALENTS FOR THE GAUSS AND OTHER ELECTROMAGNETIC UNITS

One may consider the science of practical electricity, electromagnetism, and electrostatics as a study of phenomena involving large numbers of a single fundamental physical particle, the electron. Early measurements of e/m by Thomson and of electrolysis by Faraday established such "atomicity." The individual electron was found to possess a "charge" of 4.803×10^{-10} statcoulomb, suggesting that the familiar esu unit contains about $(4.803 \times 10^{-10})^{-1}$ or 2.08×10^{9} electrons (**7, 8**). The corresponding practical electrical unit, the coulomb, was similarly found to contain about 6.2×10^{18} electrons, that is, 3×10^{9} esu's. Finally, the electromagnetic or "emu" unit, the abcoulomb, was found equivalent to 3×10^{10} esu. (Note that this multiplier is equal to the coefficient for the fundamental constant of light velocity c, that is, 3×10^{10} cm sec^{-1}.)

Since the electromagnetic unit of *charge* can be interpreted from the e/m ratio to represent a number of electrons $N = 6.2 \times 10^{19}$, each having

[2] The bold face numbers in parentheses refer to the references appended to this paper.

a rest mass $m_e = 9.106 \times 10^{-28}$ g, it is possible to consider that

$$(Q_e): \text{one abcoulomb} = Nm_e = 5.684 \times 10^{-8}\ \text{g}. \tag{1}$$

In the experimental definition of units, it is customary to employ a separation or radius $r = 1.0$ cm., a length $l = 1.0$ cm., and a time $t = 1.0$ sec. It is then possible from familiar equations to derive a table of similar cgs equivalents for other emu's as follows:

$$(I_e): \text{one abamp} = \frac{\text{abcoulomb}}{\text{sec}} = \frac{Nm_e}{t} = 5.684 \times 10^{-8}\,\frac{\text{g}}{\text{sec}} \tag{2}$$

$$(E): \text{one erg} = \text{dyne} \times 1 = \text{abwatt} \times t = 1.0\,\frac{\text{g cm}^2}{\text{sec}^2} \tag{3}$$

$$(F)^*: \text{one dyne} = \frac{\text{erg}}{1} = \frac{\text{abwatt} \times t}{1} = 1.0\,\frac{\text{g cm}}{\text{sec}^2} \tag{4}$$

$$(P): \text{one abwatt} = \frac{\text{erg}}{t} = \frac{\text{dyne} \times 1}{t} = 1.0\,\frac{\text{g cm}^2}{\text{sec}^3} \tag{5}$$

$$(V_e): \text{one abvolt} = \frac{P}{I_e} = 1.759 \times 10^{7}\,\frac{\text{cm}^2}{\text{sec}^2} \tag{6}$$

$$(R_e): \text{one abohm} = \frac{V_e}{I_e} = 3.096 \times 10^{14}\,\frac{\text{cm}^3}{\text{g sec}} \tag{7}$$

$$(R_G): \text{one abohm. cm} = \frac{(R_e)r^2}{1} = 3.096 \times 10^{14}\,\frac{\text{cm}^3}{\text{g sec}} \tag{8}$$

$$(L_e): \text{one abhenry} = (R_e)t = 3.096 \times 10^{14}\,\frac{\text{cm}^2}{\text{g}} \tag{9}$$

$$(C_e): \text{one abfarad} = \frac{t}{(R_e)} = 3.231 \times 10^{-15}\,\frac{\text{g sec}^2}{\text{cm}^2} \tag{10}$$

$$(m_e): \text{one maxwell} = (V_e)t = 1.759 \times 10^{7}\,\frac{\text{cm}^2}{\text{sec}} \tag{11}$$

$$(\mu_e): \text{mag. perm. constant of vacuum} = \frac{(m_e)^2}{(F)r^2} = 3.096 \times 10^{14}\,\frac{\text{cm}}{\text{g}} \tag{12}$$

$$(M_e): \text{unit mag. moment} = (m_e)1 = 1.759 \times 10^{7}\,\frac{\text{cm}^3}{\text{sec}} \tag{13}$$

$$(H_e): \text{one oersted} = \frac{(I_e)1}{r^2} = 5.684 \times 10^{-8}\,\frac{\text{g}}{\text{cm sec}} \tag{14}$$

* Engineers will note that the gram of *mass* has been chosen as fundamental. This gm-force unit neglects the units of g_e in the definition of the dyne, where force is conventionally symbolized by F_{g_e}.

$$(B_g)\text{: one gauss} = (\mu_g H_g) = \frac{1.759 \times 10^7}{\text{sec}} \tag{15}$$

Thus, it is possible to write uncomplicated, purely dimensional, and quantitative "mechanical equivalents" for every unit and constant employed in the measurement of electricity and magnetism. Note that coefficients for each of the above emu equivalents could be converted to their esu or practical electrical counterparts by well-known numerical differences between the electromagnetic, electrostatic, and practical electrical systems of measurement. It will also be noted that when the electron's rest mass is thus considered as a fundamental parameter in the phenomena of electromagnetism, the free-space electronic constant of magnetic permeability, μ_g (conventionally ascribed a "value" of $\mu_0 = 1.0$), acquires a quantitative coefficient and dimensions (12 above) in much the same classical sense that G, c, and $\hbar$ have them. It is interesting and significant to observe that the above expression for μ_g, derived by the simplest of logic, can also be alternately derived with great exactness by a simple interrelationship of well-known and highly important physical quantities,[3] that is,

$$\mu_g = \frac{h\alpha_e}{m_e{}^2 c} = 3.096 \times 10^{14} \frac{\text{cm.}}{\text{g}} \tag{12}$$

Knowing that $\epsilon_0 = 1/\mu_0 c^2$ (8), it is then significant to consider the dielectric constant of free-space, ϵ_g (conventionally ascribed a "value" of $\epsilon_0 = 1.0$), as a quantity equal to

$$\epsilon_g = \frac{m_e{}^2}{h\alpha_e c} = \frac{1}{\mu_0 c^2} = 3.595 \times 10^{-36} \frac{\text{g sec}^2}{\text{cm}^3}. \tag{16}$$

Note that $\epsilon_g{}^{-1}$ thus acquires dimensions similar to the gravitational constant G; however, the coefficient is some 4.15×10^{42} greater, consistent with well-known differences of binding energies of gravitation and electrostatics (6). It was also of considerable interest to note that the quantity (14) above for the oersted unit permitted the development of an exact dimensional quantity for the Bohr magneton of electron magnetic moment (9, 10), that is,

$$\text{Bohr magneton}_g = \frac{9.274 \times 10^{-21} \text{ ergs}}{(\text{oersted}_g)} = 1.632 \times 10^{-13} \frac{\text{cm}^3}{\text{sec}}. \tag{17}$$

Note, as such, that it acquires dimensions consistent with those revealed in quantity (13). Note also that its simple coefficient and dimensional value may be alternately derived from fundamental quantities, that is,

$$\text{Bohr magneton}_g = \frac{\hbar^2 \alpha_e}{2 m_e{}^2 c} = 1.632 \times 10^{-13} \frac{\text{cm}^3}{\text{sec}}. \tag{18}$$

[3] See subsequent discussions of α_e and $\hbar$.

These novel mechanical equivalents of electrical and magnetic phenomena created an interest in the possibility that mechanical interpretations for all types of physical systems might be attained. Subsequent discussions provide examples to suggest how the equivalences (1) through (18) might be employed in "classical mechanical interpretations" of either astronomic or submicroscopic phenomena.

TERRESTRIAL AND STELLAR MAGNETISM: BLACKETT'S HYPOTHESIS

During the last decade P. M. S. Blackett (3, 4) has hypothesized that the magnetism associated with the earth and other astronomic bodies is somehow related to the angular momentum of those bodies. His analysis of several bodies (for which mass, volume, and intensity of magnetization had been measured) indicated that a proportionality constant existed between the angular momenta and the magnetic moments (and/or the intensities of magnetization) of all those bodies. If his hypothesis could be proven correct, it might be assumed that the above constant of proportionality would have physical and philosophical implications similar to those embodied in Newton's constant G of gravitation.

Taking note that the earth is characterized by a magnetic field of about 0.5 gauss (or 0.5 maxwell per cm^2), rather uniformly permeating its entire spherical surface and being oriented approximately at 90° to its plane of rotation, it appeared of real value to measure that field as a cgs quantity made possible by (15) above, for example,

$$B_g \text{ (earth's field)} \cong \frac{8.77 \times 10^6}{\text{sec}}.$$

Similar conversions were made for the fields of other bodies studied and reported by Blackett. Since the new dimensions for intensity of magnetization suggested a quality of vibration or "frequency," it was decided to symbolize such a phenomenon by ν_m, for example, for the earth,

$$\nu_m = B_g = \frac{8.77 \times 10^6}{\text{sec}}.$$

In line with such symbology, it was subsequently realized that where λ_r is a measure of the solid body's axial radius in cm, ν_r is its rotational angular velocity in radians.sec^{-1}, and M_n is its rest mass in grams, the magnetic fields of the bodies studied by Blackett could be very closely approximated by

$$\nu_m = \frac{\mu_b M_n \nu_r}{\lambda_r},$$

where μ_b could be formulated by, and had a value of,

$$\mu_b = \frac{\lambda_r \nu_m}{M_n \nu_r} = 1.295 \times 10^{-8} \frac{\text{cm}}{\text{g}}. \tag{19}$$

It thus represented a somewhat startling constancy in the interrelationships of the four significant physical parameters of rotating astronomic bodies. Though the above evaluation is not precisely that suggested by Blackett, inspection will reveal that it essentially reflects and supports his hypothesis. Thus μ_b becomes a quantitative and dimensional definition of the constant of proportionality he discovered.

Noting the unique symbolism and the formula of μ_b, perhaps a new constant of classical mechanics or astronomy, it is significant to point out an interesting analogous feature of the gravitational constant G, as follows: where λ_0 is the radius of a given satellite's orbit in cm., ν_0 is the angular velocity of that orbit in radians.sec^{-1}, and M_n is the rest mass of the central body in grams, it is found universally that

$$G = \frac{\lambda_0^3 \nu_0^2}{M_n} = 6.67 \times 10^{-8} \frac{\text{cm}^3}{\text{g sec}^2}, \tag{20}$$

or 6.67×10^{-8} dyne cm^2 per gram^2. Though this interesting fact should be apparent from knowledge of the Harmonic Law and the use of G in the mathematics of gravitational force, it is not generally recognized or stressed in the teaching of classical mechanics.

Since M_n is common to the formulae of both μ_b and G, it is of further interest to point out the possibility that all mass-space-time (and/or magnetic) parameters of an astronomic solid body and its associated planetary system may then for formulated as follows:

$$F_g = \frac{G}{\mu_b^2} = \frac{M_n \lambda_0^3 \nu_0^2 \nu_r^2}{\lambda_r^2 \nu_m^2} = 3.978 \times 10^8 \frac{\text{g cm}}{\text{sec}^2}, \tag{21}$$

where F_g has the dimensions of a "force" which appears to characterize the universe. Such a force may be described as "mechanical-magnetic" in nature. As such, the quantity F_g can be used to correlate the gravitational and magnetic phenomena of astronomic (or classical mechanical) systems and "particles."

INTERRELATIONSHIPS OF PHYSICAL CONSTANTS, LAWS, AND HYPOTHESES

Having noted the development of μ_b as a universal constant of "astronomic magnetism," the implications of its formula, and its relation to G or F_g, it is significant to note also that

$$\frac{m_e^3 c^3}{\alpha_e^3 h} = 3.978 \times 10^8 \frac{\text{g cm}}{\text{sec}^2}, \tag{22}$$

which suggests that F_g is also a constant of submicroscopic physics. Note that m_e is the well-known electronic rest mass (9.106×10^{-28} g),

α_e is the dimensionless electronic fine structure constant (7.298×10^{-3}), c is the constant of light velocity (2.998×10^{10} cm sec^{-1}), and h is the constant of angular momentum (1.054×10^{-27} g cm^2 sec^{-1}). Since m_e and α_e are properties of a specific submicroscopic particle (the electron), the above formula for F_β would suggest that the quantity $m_e\alpha_e^{-1}$ might represent a *general* physical constant of all submicroscopic particles. From considerations of the Bohr magneton, as presented in quantities (17) and (18) above, and from suggestions inherent in quantities (12) and (16), it is believed that

$$M_s = \frac{m_e}{\alpha_e} = \frac{M_n}{\alpha_n} = \frac{M_n\nu_r}{\nu_m} = 1.248 \times 10^{-25} \text{ g}, \tag{23}$$

where M_s is then considered a significant constant of submicroscopic physics. Thus, in a manner somewhat analogous to the formulation of μ_b in quantity (19), the above formula suggests a constancy in the free-space interrelationship of any submicroscopic particle's properties of rest mass M_n in grams, its angular velocity ν_r in radians.sec^{-1}, and its intensity of magnetization ν_m in sec^{-1} units. (The angular velocity ν_r of the rotating submicroscopic particle M_n at a given energy E is assumed to be known from $\nu_r = Eh^{-1}$, that is, it is equivalent to 2π times its radiation frequency.) Thus M_s is, in a sense, a philosophical and physical analogy to the constant μ_b of astronomic systems, that is, it serves to relate the body's magnetic field ν_m to its properties of angular motion.

It is of further interest to note that the submicroscopic particle M_n at a given energy E (and/or angular velocity ν_r) will also possess an "effective" or deBroglie radius λ_r equal to $(h^2M_n^{-1}E^{-1})^{\frac{1}{2}}$ centimeter. Note that such a radius would be equivalent to the particle's deBroglie wave-length (translational) divided by 2π. Thus it can be shown that the universal free-space constant of angular momentum can be formulated by

$$h = M_n\lambda_r^2\nu_r = 1.054 \times 10^{-27} \frac{\text{g cm}^2}{\text{sec}}, \tag{24}$$

or 1.054×10^{-27} erg.sec. Though this formula should be intuitively known, it is seldom emphasized in the purely mathematical use of h in physical calculations. The particle M_n in free-space motion at a given energy E may also be considered to possess a "light" particle as a satellite at an "orbital" radius $\lambda_0 = hc/E$ centimeter. Such a radius would be equivalent to the characteristic minimum wave-length (of light radiation) divided by 2π. The angular velocity ν_0 of such an orbit will be recognized as equal to M_n's rotational ν_r at the given energy. Thus, the free-space particle M_n will be characterized by space-time parameters which have the universally constant relationships

$$R_g = \frac{\nu_r}{\nu_0} = 1.000, \tag{25}$$

and

$$c = \lambda_0 \nu_0 = 2.998 \times 10^{10} \frac{\text{cm}}{\text{sec}}. \tag{26}$$

(In order to substantiate the fundamental implications previously attributed to the quantity (18), and/or the quantity (23), it may now be shown that an electron at any energy will always possess a magnetic moment of $\lambda_r^4 \nu_m / 2\lambda_0 = h^2 \alpha_e / 2 m_e^2 c = h^2 / 2 M_e c m_e = 1.632 \times 10^{-13}$ cm^3 sec^{-1}.)

Referring back to the quantity (22), and the related implications of quantity (23), it will then be recognized that the universal constant F_g of submicroscopic physics can be formulated as follows:

$$F_g = \frac{m_e^2 c^3}{\alpha_e^2 h} = \frac{M_e^2 c^3 R_g}{h} = \frac{M_n \lambda_0^3 \nu_0^3 \nu_r^2}{\lambda_r^3 \nu_m^2} = 3.978 \times 10^8 \frac{\text{g cm}}{\text{sec}^2}. \tag{27}$$

Combining quantities (21) and (27), a significant correlation of all the mechanical and magnetic aspects of *both* submicroscopic and astronomic particles may be realized, that is,

$$F_g = \frac{G}{\mu_0^2} = \frac{M_e^2 c^3 R_g}{h} = \frac{M_n \lambda_0^3 \nu_0^3 \nu_r^2}{\lambda_r^2 \nu_m^2} = 3.978 \times 10^8 \frac{\text{g cm}}{\text{sec}^2}. \tag{28}$$

Thus F_g appears to be a "truly universal" constant of mass-space-time in all natural systems of mechanical and magnetic interactions.

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APPENDIX A

The accompanying Table I of cgs data has been prepared as a convenience for the reviewer

TABLE I.

	Submicroscopic Particles		Astronomic Particles	
Symbol	13.54 ev Electron	7.2 Mev Neutron	Earth (Moon's Orbit)	Sun (Earth's Orbit)
M_n	9.1×10^{-28} g	1.67×10^{-24} g	5.98×10^{27} g	1.97×10^{33} g
λ_0	1.45×10^{-6} cm	2.74×10^{-13} cm	3.84×10^{10} cm	1.49×10^{13} cm
ν_0	2.07×10^{16} sec^{-1}	1.1×10^{23} sec^{-1}	2.66×10^{-6} sec^{-1}	2.0×10^{-7} sec^{-1}
λ_r	7.46×10^{-9} cm	2.4×10^{-13} cm	6.36×10^{8} cm	6.95×10^{10} cm
ν_r	2.07×10^{16} sec^{-1}	1.1×10^{23} sec^{-1}	7.29×10^{-6} sec^{-1}	2.9×10^{-6} sec^{-1}
ν_m	1.52×10^{14} sec^{-1}	1.47×10^{23} sec^{-1}	8.86×10^{6} sec^{-1}	1.06×10^{9} sec^{-1}
	Note by substitution that:		Note by substitution that:	
	$M_n\lambda_r^2\nu_r = 1.054 \times 10^{-27} \frac{\text{g cm}^2}{\text{sec}} = \hbar$		$\frac{\lambda_0^3\nu_0^2}{M_n} = 6.67 \times 10^{-8} \frac{\text{cm}^3}{\text{g sec}^2} = G$	
	$\frac{M_n\nu_r}{\nu_m} = 1.248 \times 10^{-24}\,\text{g} = M_e$			
	$\lambda_0\nu_0 = 3 \times 10^{10} \frac{\text{cm}}{\text{sec}} = c$		$\frac{\lambda_r\nu_m}{M_n\nu_r} = 1.29 \times 10^{-8} \frac{\text{cm}}{\text{g}} = \mu_0$	

Note in all four cases that:

$$\frac{M_n\lambda_0^3\nu_0^2\nu_r^2}{\lambda_r^2\nu_m^2} = 3.987 \times 10^{8} \frac{\text{g cm}}{\text{sec}^2} = F_g$$

in checking the numerical coefficients and dimensions of various quantities and formulae presented in the paper. Definitions of the symbols and the meanings (or methods for determination) of the associated quantities were given in the discussion.

The four familiar physical particles were chosen (1) because it was felt that the reader would more readily perceive the cgs magnitudes in relation to other unit systems, and (2) because certain of the data will be employed in familiar examples to illustrate the possible significance of new quantities revealed in the study. The λ_0 value for the 13.54 ev hydrogen electron will be recognized as the well-known Lyman wave-length (9.11×10^{-6} cm) divided by 2π, whereas the particular λ_0 value for the Sun is the average distance (9.29×10^{7} miles $\times 1.61 \times 10^{5}$ cm/mile) to the Earth's orbit. The ν_0 of the electron is recognized as equal to c/λ_0, or $E\hbar^{-1}$, where E is equal to 2.18×10^{-11} g cm^{2}sec^{-2} or 13.54 ev. The fact that its $\nu_0 = \nu_r$ is not unparalleled in classical mechanics, since this is known to be the case for both the Moon's and Mercury's rotational (ν_r) and revolutional (ν_0) "frequencies." The ν_0 for the Sun will be recognized as one revolution or 2π radians per year $\times 3.17 \times 10^{-8}$ years/second, that is, descriptive of the earth's orbit "frequency." Note that where data for any of the several planetary orbits would be considered, the value of the product $\lambda_0^3\nu_0^2$ for the Sun would be a constant. In other words, any given central body of mass M_n can be characterized by several values of λ_0 and a corresponding ν_0, and the several "harmonic relations" of $\lambda_0^3\nu_0^2$ are constant.

The λ_r for the electron will be observed to closely approximate the reported radius of the hydrogen atom. Similarly, λ_r for the neutron at 7.2 Mev (the approximate binding energy per nucleon in the nucleus of any atom) is roughly equal to the distance between the nucleons, and/or the estimated neutron radius. The λ_r for the earth will be observed as equal to its 3.950 mile axial radius in centimeters, and its rotational ν_r is equal to one rotation or 2π radians per day $\times$ 1.16×10^{-5} days/second. Based on conversions made possible by knowledge of quantity (15) in the discussion, the ν_m's for the Sun and Earth will be found equal to 60 gauss and 0.5 gauss, respectively.

APPENDIX B

Because of its fundamental importance in the mathematics of spectral analysis, the 13.54 ev energy or ionization potential of the hydrogen electron is designated as a unit of energy, often called the "Rydberg constant." The radius of the corresponding orbit is considered to be $r = 5.29 \times 10^{-9}$ cm; so, where conventionally e^- designates the specific electronic charge (4.8×10^{-10} esu), e^+ an equivalent specific positive charge on the proton (and/or the positron), and $\epsilon_0 = 1.0$ the dielectric constant of free-space, the Rydberg energy can be determined by

$$E = \frac{e^+e^-}{2\epsilon_0 r} = 2.18 \times 10^{-11} \frac{\text{g cm}^2}{\text{sec}^2} = 13.54 \text{ ev.}$$

From quantities described in the discussion, it will also be found that this energy can be alternately obtained by:

$$E = \frac{\hbar\alpha_s c}{2r} = \frac{m_s{}^+m_s{}^-}{2\epsilon_0 r} = \frac{m_s{}^+m_s{}^-\mu_0 c^2}{2r} = \frac{m_s \hbar c}{2rM_s} = \frac{M_s{}^2c^2\alpha_s}{2rF_g} = \frac{\hbar^2}{2m_s r^2}$$

$$= 2.18 \times 10^{-11} \frac{\text{g cm}^2}{\text{sec}^2} = 13.54 \text{ ev.}$$

Such equivalences effectively point out the interrelationships existing between the several various quantities of electromagnetism and quantum mechanics. From the electron data in the above table, it is of further interest to note also that

$$E = \frac{\hbar^2}{m_e \lambda_r{}^2} = \frac{\hbar c}{\lambda_0} = \frac{P^+P^-F_g}{\lambda_0 c^2} = \frac{(M_n M_g)_e{}^2 F_g}{\lambda_0 \hbar^2} = \frac{(M_n M_g)_e (M_n M_g)_n F_g}{\lambda_0 \hbar^2}$$

$$= 2.18 \times 10^{-11} \frac{\text{g cm}^2}{\text{sec}^2} = 13.54 \text{ ev}$$

where (a) the quantity P refers to a constant "magnetic pole" (presumably associated with the electron or any submicroscopic particle) equal to $\hbar M_e{}^{-1} = \lambda_r{}^2 \nu_m = 8.45 \times 10^{-3}$ cm^3 sec^{-1}, (b) the quantity M_{ge} is considered as the electron's magnetic moment of 3.26×10^{-18} cm^3 sec^{-1} or 2.0 Bohr magnetons, and (c) the quantity M_{gn} is the neutron's magnetic moment of 1.7×10^{-16} cm^3 sec^{-1} or 1.913 nuclear magnetons. (Note that these two magnetic moments may be determined from tabulated data as $\lambda_r{}^4 \nu_m \lambda_0{}^{-1}$.) Thus, the last expression in the equation suggests a simultaneous interaction of the masses (M_n) and magnetic moments (M_g) of the electronic and neutronic particles. A somewhat analogous situation in the interaction of astronomic particles is described in Appendix C.

APPENDIX C

From data given in the above table it will be noted that the gravitational "binding" energy in the Sun-Earth system may be conventionally determined by

$$E = \frac{M_n M_1 G}{\lambda_0} = 5.26 \times 10^{40} \frac{\text{g cm}^2}{\text{sec}^2},$$

where M_1 designates the earth's mass, and where M_n and λ_0 symbols characterize the Sun. From the formula for G, the equation can then be alternately written as

$$E = \frac{M_n{}^+ M_1{}^-}{\lambda_0} \left(\frac{\lambda_0{}^3 \nu_0{}^2}{M_n} \right) = M_1 \lambda_0{}^3 \nu_0{}^2 = 5.26 \times 10^{40} \frac{\text{g cm}^2}{\text{sec}^2},$$

signifying that $\lambda_0 \nu_0$ is the peripheral velocity of the earth in its orbit about the Sun, and that its mass × velocity^2 is equivalent to the "gravitational" energy. (Note that the gravitational force would be equal to the energy divided by λ_0.) Since the gravitational energy is considered to be a phenomenon of attraction, the + and − signs for mass were employed.

It is of further interest to note from tabulated data that, where $(\lambda_r \nu_m \nu_r^{-1})_n$ and $(\lambda_r \nu_m \nu_r^{-1})_1$ for the Sun and the Earth are symbolized as L_n and L_1 respectively, the binding energy E can be alternately determined by

$$E = \frac{L_n + L_1 + F_g}{\lambda_0} = 5.26 \times 10^{46} \frac{\text{g cm}^2}{\text{sec}^2}.$$

This is indicative that in two-body interactions the formula for F_g might be written as

$$F_g = \left(\frac{G}{\mu_0}\right)_n \left(\frac{1.0}{\mu_0}\right)_1 = \left(\frac{\lambda_0^3 \nu_0^3 \nu_r}{\lambda_r \nu_m}\right)_n \left(\frac{M_1 \nu_r}{\lambda_r \nu_m}\right)_1 = 3.978 \times 10^8 \frac{\text{g cm}}{\text{sec}^2}.$$

Thus, the above equation for E could be written

$$E = \frac{F_g}{\lambda_0} \left(\frac{\lambda_r \nu_m}{\nu_r}\right)_n^+ \left(\frac{\lambda_r \nu_m}{\nu_r}\right)_1^+ = M_1 \lambda_0^2 \nu_0^2 = 5.26 \times 10^{46} \frac{\text{g cm}^2}{\text{sec}^2}.$$

Inspection of the equation will reveal the implication that (where F_g is accepted as a constant and where the axial radii λ_r, the magnetic intensities ν_m, and the angular velocities ν_r can be measured), the gravitational and/or magnetic energy E can be determined without a knowledge of the particles' masses. Since the Sun and Earth rotate in the same direction it may be supposed that the north$^+$-south$^-$ directions of their magnetic fields ν_m are parallel and would therefore philosophically suggest dipolar repulsion and/or a positive sign for the magnetic energy E. If this were indeed true, it may be suggested that the gravitational attraction and the magnetic repulsion are balanced.

An interesting implication of the latter two equations for E, assuming that F_g is always constant, is that a change or fluctuation in the Sun's magnetic field ν_m (or its ν_r or λ_r) might impress an immediate proportional change in the Earth's magnetic field ν_m. Slower corresponding changes might also conceivably occur in its ν_r or λ_r, or perhaps in the λ_0 of separation of the two bodies. Such an implication may afford an explanation for the mysterious relationship of terrestrial magnetic cycles to the earth's position and motions relative to the Sun and/or Moon. It may similarly account for the relatively severe terrestrial magnetic disturbances which accompany high sunspot and magnetic activity. (Also, since Jupiter is the Sun's major planet and since it has an eleven$^+$ year revolutional period, it may be suggested that the approximately 11-year cycle of sunspot activity is in some manner affected by the Sun's λ_0 in relation to massive Jupiter.)

If these things be true, the "laws" implied in the constant F_g would strengthen the meaning of Kepler's eloquence when he marvelled at the mysterious "harmony of the spheres."

X-Ray Viewer Demonstrated at Medical Meeting.—An unique electronic instrument that increases readability of X-ray negatives through contrast enhancement was demonstrated during the 11th Annual Clinical meeting of the American Medical Association, Philadelphia. This was the first utilization of the Philco "EXICON" (*EX*panded *I*mage *CON*trast) X-ray viewer before any group of the AMA.

"EXICON"-enhanced X-ray negatives were televised from Lankenau Hospital to the AMA scientific sessions at Convention Nall during programs presented by the Medical Color Television Unit of the Smith, Kline and French Laboratories.

"EXICON" quickly and accurately extracts a maximum amount of information from an X-ray negative, according to Dr. J. Gershon-Cohen, Director of Radiology Department, Einstein Medical Center, Philadelphia.

He pointed out that this new electronic system enhances X-ray contrast in "gray scale" variations and magnifies the area being viewed. Dr. Gershon-Cohen was closely associated with Philco's research engineers during the development of "EXICON" at the company's Government and Industrial Division plant here.

Dr. Gershon-Cohen went on to explain that the human eye, like the majority of senses, responds to stimuli on a relative, or percentage basis. For example, the eye will recognize an area of a picture as being brighter than a neighboring area, provided there is a certain brightness differential.

"Important information contained in a small area of low contrast X-ray negatives is frequently difficult to discern by the usual visual process, even to a trained technician," he commented.

Through the application of advanced electronics and principles used in standard television techniques, "EXICON" significantly enhances fractional percentages of "gray scale" contrast to assist in positive X-ray diagnosis.

With this new medical electronic device a small area of any X-ray negative can be enlarged to a greatly expanded black-to-white contrast range. This sharp precise contrast enhancement enables more positive reading of important details in an X-ray negative.

The "EXICON" system consists of a monochrome television monitor, operators' console and a Philco flying spot scanner. An X-ray negative placed before the flying spot scanner is magnified and reproduced in enhanced monochrome. Negatives may be quickly shifted about to obtain magnified images of any portion.

The flying spot scanner converts the information on the X-ray negatives into an electrical signal in order to process the information through the contrast enhancing device in "EXICON." This flying spot of light travels an area four inches by 3/1000th of an inch in 1/40,000th of a second.

This internally generated electron beam produced on the scanner tube's screen is focused by a system of lenses upon the X-ray negative under examination.

As this bright spot of light passes through the negative, it is picked up by a photo electric tube. Then the light is converted into a video signal, processed by special electronic circuitry and fed to a TV-type monitor where the X-ray picture is reproduced for viewing.

BURSTING PRESSURES AND SAFETY FACTORS FOR THIN-WALLED VESSELS

BY

N. A. WEIL [1]

ABSTRACT

This paper deals with the bursting of vessels under slowly increasing pressures. The equations governing instability (bursting) are formulated for tensile bars and for vessels under internal pressures, assuming that the octahedral shear stress and strain uniquely describe the straining history of the material, and that the Hencky-Mises theory governs plastic behavior.

Closed form solutions are obtained by adopting a parabolic expression for the true tensile stress-strain properties of the material. The results show that the bursting stress in vessels may be either lower or higher than the ultimate tensile stress, depending upon the properties of the material. The safety factor for bursting is shown to depend solely upon the strain-hardening exponent, while the U.T.S. is proven to represent no real property of the material.

Results are compared with recent proposals advanced by the Subcommittee on Pressure Vessel Shell Theory for inclusion in the ASME Code. It is shown that a rational explanation of the proposed code equation can be advanced, provided that the thickness-diameter ratio is replaced by a factor containing the strain-hardening exponent of the material.

The solution is extended to cover bursting under a pressure rise due to adiabatic combustion, and to the case of gas volume compression from atmospheric conditions to instability. Some experimental values for material properties are also presented, followed by predictions for the stresses, strains, instability pressures, safety factors, limiting combustion pressure rise and gas volume compression at bursting for an SAE 285, Grade B steel having assumed material properties.

NOMENCLATURE

P = Tensile load, lb
A = Current cross-section in tension, in^2
L = Current gage length, in
s = Conventional stress, psi
s_{ult} = Ultimate tensile strength (U.T.S.), psi
s_{burst} = Conventional circumferential stress at maximum pressure in vessels, psi
s_{des} = Conventional design stress, psi
σ = True stress, psi
e = Conventional strain
ϵ = Logarithmic strain
p = Pressure, psi
p_a = Atmospheric pressure, psi
p_d = Design pressure, psi
t = Current wall thickness of vessel, in
r = Current radius of vessel, in
B = Hardness factor, psi
n = Strain-hardening exponent
S.F. = Design safety factor

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Q = Safety factor quotient between vessels and tensile members
V = Vessel volume per unit length, in^3
μ = Volume expansion under constant pressure adiabatic combustion
Subscripts:
x,c,r Refer to quantities acting in the axial, circumferential and radial directions.
1,2,3, Designate stresses or strains in the three principal directions.
0 Refers to initial (unstrained) dimensions.
Subscripts for σ have the same meaning as those for s.

I. INTRODUCTION

From the earliest days of rational engineering design, a quest was made to determine the mechanical properties of materials on the basis of a convenient and simple test.[2] Such a test was soon assumed to have been found in the uniaxial tensile test, which thereafter came to be universally applied to determine the design properties of materials.

The principal material property obtained from the uniaxial tensile test is the "ultimate tensile strength." The U.T.S. corresponds to the maximum load developed during the experiment, and is based on original cross sectional areas. The design stress is then obtained by the application of a safety factor (S.F.), generally 3 or 4, to the U.T.S. The assumption inherent in this approach is that a margin of safety, corresponding to the S.F., will thus become available to account for loads of an unexpected intensity or for stresses unevaluated owing to the lack of understanding of the art.

This would be a satisfactory approach, provided that its application were limited to structural members subjected to uniaxial tension only. In actual practice, however, this property is applied uncritically to setting design stress limits for pipes under applied pressures or process unit components subjected to fatigue loading under a reversal of stresses. The implication is that disruption of the structural piece, be that a tensile bar, a cylindrical pipe or a spherical container, will occur at the same stress level as represented by the U.T.S. It is for this reason that it becomes necessary to examine the nature of the U.T.S., both as to what real property it represents for materials, and its applicability to members of various geometrical shapes.

Tensile rupture occurs when the strain hardening ability of the material no longer is able to compensate for the reduction in area that occurs under plastic flow. If the load at this point is maintained with undiminished intensity, a sudden and violent rupture will take place. In the simple uniaxial tensile test a sudden failure is avoided simply because the straining ability of the test specimen outruns the speed of cross-head movement, permitting thereby the load to drop off. This is properly registered in the tensile test by a reduction of load beyond the

[2] Editor's Note: The Franklin Institute set up a committee in 1831 to study the causes of steam boiler explosions. The Committee reports were published in this JOURNAL from 1831 to 1836 and were later issued as a separate pamphlet. In the course of its investigations, the Committee developed the first apparatus to test the strength of materials. The original machine is still on display in the Institute's Museum.

maximum level (corresponding to the U.T.S.) and physical separation of the tensile coupon into two parts.

Tensile rupture is an instability phenomenon, akin to buckling in compression and differing from it only in the physical criteria that need to be applied for deriving the governing equations. Since large plastic strains are involved in tensile instability, the simple concepts of "engineering" or "conventional" stress and strain no longer suffice, and it becomes necessary to describe conditions in terms of "logarithmic strain" and "true stress," based on the changing cross sectional area. Also, the large magnitude of plastic strains as compared to the elastic component at instability makes it permissible to ignore the contribution of elastic strains to physical behavior. This is done merely for convenience of the solution; it is perfectly possible to include elastic strains in the solution. In fact, this should be done any time that the plastic instability strain is small, say less than 5 per cent, as in brittle (purely elastic) or greatly strain-hardened or temper-hardened materials. In the following treatment of the rupture problem, the material will be assumed to have sufficient ductility to reach the instability strain without prior fracture.

II. INSTABILITY IN UNIAXIAL TENSION

In a uniaxial tensile test the conventional stress-strain ($s - e$) diagram for metals has generally the shape shown in Fig. 1. The con-

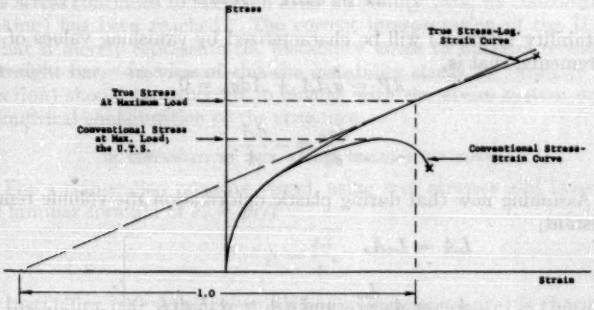

FIG. 1. Stress-strain curves in tension.

ventional stress and strain are defined as

$$s_z = \frac{P}{A_0} \tag{1}$$

$$e_z = \frac{L - L_0}{L_0} = \frac{L}{L_0} - 1. \tag{2}$$

Instead of the quantities in Eqs. 1 and 2 one may use the true stress and logarithmic strain defined as

$$\sigma_z = \frac{P}{A} \tag{3}$$

$$\epsilon_z = \ln \frac{L}{L_0} = \ln (1 + e_z). \tag{4}$$

The true stress-logarithmic strain diagram is also shown in Fig. 1. It is noteworthy to observe that, unlike the conventional curve which exhibits a maximum followed by a drop-off in stress, the true stress-strain curve is a monotonously increasing one, bespeaking continued strain-hardening all the way to fracture. In fact, no condition of strain-softening (as would be implied beyond the U.T.S. by the conventional curve) has ever been noted in carefully conducted experiments.

The instability (maximum load) condition is easily recognized for the conventional stress-strain curve of Fig. 1; it occurs at the highest horizontal tangent of this curve. However, conventional stresses are not easily transferred from one loading condition to another. For this reason, until the conclusion of these comments, further treatment will be concerned with true stresses and logarithmic strains.

By Eq. 3, $P = \sigma_z A$, so that increments of P can be expressed as

$$dP = \sigma_z dA + A d\sigma_z.$$

Instability (P max.) will be characterized by vanishing values of load increments, that is,

$$dP = \sigma_z dA + A d\sigma_z = 0$$

or

$$\frac{d\sigma_z}{\sigma_z} = -\frac{dA}{A}. \tag{5}$$

Assuming now that during plastic deformation the volume remains constant,

$$\left.\begin{aligned} LA &= L_0 A_0 \\ \text{or} \\ A &= A_0 \frac{L_0}{L} \quad \text{and} \quad dA = A_0 L_0 d(L^{-1}) \end{aligned}\right\}. \tag{6}$$

Substituting this result into Eq. 5,

$$\frac{d\sigma_z}{\sigma_z} = -L d(L^{-1}). \tag{7}$$

However, from Eq. 4

$$L = L_0 \exp(\epsilon_z)$$

and

$$d(L^{-1}) = -\exp(-\epsilon_z)d\epsilon_z/L_0.$$

Substituting these results into Eq. 7 yields

$$\frac{d\sigma_z}{\sigma_z} = d\epsilon_z$$

or

$$\frac{d\sigma_z}{d\epsilon_z} = \sigma_z. \tag{8}$$

Equation 8 implies that the maximum load condition is reached when the tangent of the true stress-log strain curve ($d\sigma/d\epsilon$) has a slope whose value is σ: 1. The graphical representation of this concept, called the Considère construction, is also shown in Fig. 1. The Considère construction (1)[3] is equivalent to the determination of the U.T.S., except that an inclined tangent is substituted for the horizontal one denoting the maximum point in the conventional curve. Accordingly, the U.T.S. and the true instability stress are related to one another by

$$\sigma_z = s_z(1 + e_z) \tag{9}$$

as obtained from Eqs. 1, 3, 4 and 6.

Figure 1 (or Eq. 9) also shows that s_{ult} represents no fundamental material property. This quantity is not a maximum stress since the true stress continues to rise even after instability (and its concomitant, necking) has been reached. The correct interpretation of the U.T.S. is that it merely represents the instability stress in uniaxial tension of a straight bar. In view of this the instability stress (or ultimate load criterion) should be expected to change with the stress system or the geometrical configuration of the structure.

III. INSTABILITY IN THIN WALLED CIRCULAR CYLINDERS

For a thin-walled pressure vessel, using true stresses and inverting the familiar formula of $\sigma_c = pr/t$

$$p = \frac{\sigma_c t}{r}. \tag{10}$$

Instability (the attainment of the maximum pressure) is characterized by

$$dp = \frac{t}{r}d\sigma_c + \frac{\sigma_c}{r}dt - \sigma_c t\frac{dr}{r^2} = 0$$

or

$$\frac{r}{\sigma_c}\frac{d\sigma_c}{dr} + \frac{r}{t}\frac{dt}{dr} = 1. \tag{11}$$

[3] The boldface numbers in parentheses refer to the references appended to this paper.

Volume constancy for a pressure vessel (assuming no axial strain, as will be shown later) demands that

$$rt = r_0 t_0. \tag{12}$$

Hence, it follows that

$$\frac{r}{t}\frac{dt}{dr} = -1; \quad \frac{r}{dr} = -\frac{t}{dt}.$$

Insertion of these results into Eq. 11 yields

$$-\frac{t}{\sigma_e}\frac{d\sigma_e}{dt} = 2$$

or

$$\frac{d\sigma_e}{\sigma_e} = -2\frac{dt}{t}. \tag{13}$$

Using now a definition for logarithmic strain identical with the one in the uniaxial case,

$$\left.\begin{aligned} \epsilon_r &= \ln\frac{t}{t_0}; \quad d\epsilon_r = \frac{dt}{t} \\ \epsilon_e &= \ln\frac{r}{r_0}; \quad d\epsilon_e = \frac{dr}{r} = -d\epsilon_r \end{aligned}\right\} \tag{14}$$

Equation 13 can be rearranged to the form:

$$\frac{d\sigma_e}{d\epsilon_r} = -2\sigma_e. \tag{15}$$

Much of the derivation presented to this point can also be found (with a somewhat different approach) in (2).

IV. INSTABILITY BASED ON THE EFFECTIVE STRESS-STRAIN CURVE

The instability condition for thin-walled tubes incorporated in Eq. 15 is not directly amenable to graphical representation. Nor can it be compared in a straightforward fashion with Eq. 7, the condition governing the instability of tensile bars.

In order to do this, additional concepts must be introduced. Assume that, in plastic flow, the relationship between stresses and strains is adequately described by

$$\left.\begin{aligned} \delta\epsilon_1 &= \delta\lambda\left[\sigma_1 - \tfrac{1}{2}(\sigma_2 + \sigma_3)\right] \\ \delta\epsilon_2 &= \delta\lambda\left[\sigma_2 - \tfrac{1}{2}(\sigma_3 + \sigma_1)\right] \\ \delta\epsilon_3 &= \delta\lambda\left[\sigma_3 - \tfrac{1}{2}(\sigma_1 + \sigma_2)\right] \end{aligned}\right\} \tag{16}$$

and that an effective measure of stress and strain is given by[4]

$$\sigma_{eff} = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{\frac{1}{2}} \tag{17}$$

$$\epsilon_{eff} = \sqrt{2}\left[(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2\right]^{\frac{1}{2}}. \tag{18}$$

Equations 16 represent the postulates of the Levy-Mises "incremental strain" theory. For stress systems where the directions of principal stresses remain fixed during deformation (which is the case considered here) they may be simplified by integration to the expressions of the Hencky-Mises "deformation" theory

$$\left.\begin{aligned} \epsilon_1 &= \lambda\left[\sigma_1 - \tfrac{1}{2}(\sigma_2 + \sigma_3)\right] \\ \epsilon_2 &= \lambda\left[\sigma_2 - \tfrac{1}{2}(\sigma_3 + \sigma_1)\right] \\ \epsilon_3 &= \lambda\left[\sigma_3 - \tfrac{1}{2}(\sigma_1 + \sigma_2)\right] \end{aligned}\right\}. \tag{19}$$

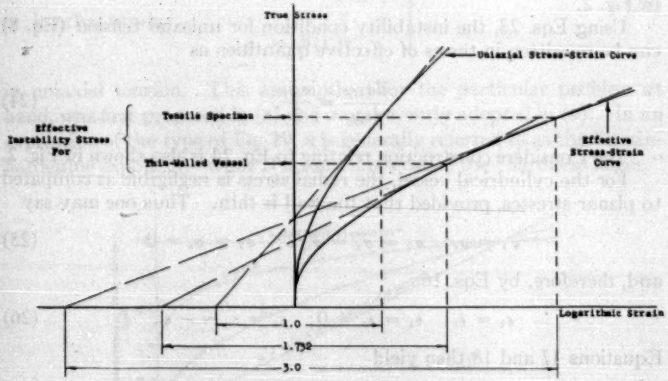

FIG. 2. The uniaxial and effective stress-strain curves.

The one further postulate to be made is that the relationship describing the universal stress-strain curve

$$\sigma_{eff} = f(\epsilon_{eff}) \tag{20}$$

should always govern material behavior, regardless of the manner of straining employed. While experiments (3, 4, 5) have shown that the von Mises theory is not rigorously correct, nonetheless Eqs. 19 and 20

[4] Equations 17 and 18 represent the "octahedral shear" stress and strain, respectively, provided that both equations are multiplied by a constant of 2/3. The adoption of unusual constants is done here only to simplify some of the numerical results derived; this choice has no bearing on the final results obtained in the paper.

satisfactorily describe the general behavior of metals in the plastic range.

With these assumptions, the stress conditions for uniaxial tension are given by

$$\sigma_1 = \sigma_x; \quad \sigma_2 = \sigma_3 = 0. \tag{21}$$

Hence, from Eqs. 19

$$\epsilon_1 = \epsilon_x; \quad \epsilon_2 = \epsilon_3 = -\epsilon_1/2. \tag{22}$$

Substitution of these results into Eqs. 17 and 18 results in

$$\begin{aligned} \sigma_{eff} &= \sigma_x \\ \epsilon_{eff} &= 3\epsilon_x. \end{aligned} \tag{23}$$

This result implies that the "effective" stress-strain curve of Eq. 20 can be obtained from the uniaxial true stress-log strain curve simply by multiplying the abscissae of the latter by a factor of 3. This is shown in Fig. 2.

Using Eqs. 23, the instability condition for uniaxial tension (Eq. 8) can be rewritten in terms of effective quantities as

$$\frac{d\sigma_{eff}}{d\epsilon_{eff}} = \frac{\sigma_{eff}}{3}. \tag{24}$$

The Considère construction relating to Eq. 24 is also shown in Fig. 2.

For the cylindrical vessel, the radial stress is negligible as compared to planar stresses, provided that the wall is thin. Thus one may say

$$\sigma_1 = \sigma_c; \quad \sigma_2 = \sigma_x = \sigma_c/2; \quad \sigma_3 = \sigma_r = 0 \tag{25}$$

and, therefore, by Eqs. 16

$$\epsilon_1 = \epsilon_c; \quad \epsilon_2 = \epsilon_x = 0; \quad \epsilon_3 = \epsilon_r = -\epsilon_c. \tag{26}$$

Equations 17 and 18 then yield

$$\left.\begin{aligned} \sigma_{eff} &= \frac{\sqrt{3}}{2}\sigma_c \\ \epsilon_{eff} &= \sqrt{12}\,\epsilon_c = 2\sqrt{3}\,\epsilon_c \end{aligned}\right\}. \tag{27}$$

Utilization of Eqs. 27 permits a reformulation of the governing equation for the instability (maximum pressure) of thin-walled tubes (Eq. 15) to the following form:

$$\frac{d\sigma_{eff}}{d\epsilon_{eff}} = \frac{1}{\sqrt{3}}\sigma_{eff} = \frac{\sigma_{eff}}{1.732}. \tag{28}$$

The Considère construction for the instability of thin-walled tubes is also indicated in Fig. 2.

Equations 24 and 28, governing the instability (limit design or, by projection, rupture) conditions of tensile specimens and thin-walled tubes, respectively, are directly comparable. This comparison, or a glance at Fig. 2, will show that the effective stress and strain values characterizing the instability of thin-walled tubes can never reach the values associated with the instability of members in pure tension.

V. RUPTURE WITH STRAIN-HARDENING AT A PARABOLIC RATE

To bring this comparison to a more tangible level, assume that the effective stress-strain curve, Eq. 20, can be given in some explicit analytical form. A very adequate, yet mathematically simple, approach is to assume that Eq. 20 has the form

$$\left.\begin{aligned} \sigma_{eff} &= B\left(\frac{\epsilon_{eff}}{3}\right)^n \\ \text{or, in other words} \qquad \sigma_z &= B(\epsilon_z)^n \end{aligned}\right\} \tag{29}$$

in uniaxial tension. This assumption, for the particular problem at hand, was first proposed in (8) and more recently adopted in (9). In an expression of the type of Eq. 29, n is generally referred to as the "strain-hardening exponent," while B is sometimes called the "hardness factor."

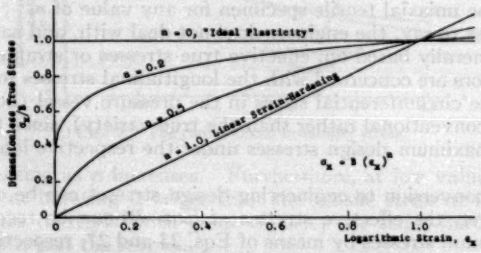

FIG. 3. Analytical representation of the tensile stress-strain curve.

The shape of this stress-strain curve for various values of n is shown in Fig. 3.[5]

On the basis of Eq. 29, explicit answers can be obtained from Eqs.

[5] Any of the curves in Fig. 3 may be raised or lowered by a suitable modification of the value of B. For materials with a large elastic range and comparatively little plastic ductility, the stress-strain curve would be better fitted by an expression of the type (13)

$$\sigma_z = A\epsilon^m + B\epsilon^n \tag{29'}$$

24 and 28 for the maximum load or pressure conditions investigated here. There results

$$\left.\begin{array}{ll} \textit{Uniaxial Tension} & \textit{Thin-Walled Tube} \\ \epsilon_{eff} = 3n & \epsilon_{eff} = \sqrt{3}n \\ \sigma_{eff}/B = n^n & \sigma_{eff}/B = \left(\dfrac{n}{\sqrt{3}}\right)^n \end{array}\right\}. \tag{30}$$

A comparison of the effective instability stresses for tensile bar and thin-walled tube is shown in Fig. 4 for various values of n. As noted

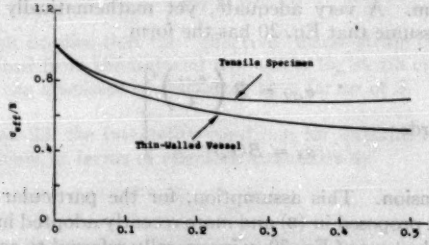

FIG. 4. The effective instability stress.

before, the effective rupture stresses are lower for the cylindrical vessel than for the uniaxial tensile specimen for any value of n.

Needless to say, the engineer does not deal with, and safety factors are not generally based on, effective true stresses or strains. Instead, safety factors are concerned with the longitudinal stresses in the tensile bar and the circumferential stress in the pressure vessel (both of these being the conventional rather than the true variety), since these represent the maximum design stresses under the respective loading conditions.

The reconversion to engineering design stresses can be done in two steps. First, the effective stresses of Eqs. 30 can be reconverted to true maximum stresses by means of Eqs. 23 and 27, respectively. The result is

$$\left.\begin{array}{ll} \textit{Uniaxial Tension} & \textit{Thin-Walled Vessel} \\ \epsilon_z = n & \epsilon_c = \dfrac{n}{2} \\ \sigma_{ult}/B = n^n & \sigma_{burst}/B = \dfrac{2}{\sqrt{3}}\left(\dfrac{n}{\sqrt{3}}\right)^n \end{array}\right\}. \tag{31}$$

The second step is to reduce true stresses to conventional stresses. This can be done by using Eqs. 4 and 9 for the uniaxial case. For the

thin-walled tube Eqs. 10, 14 and 26 provide the required information. Thus

$$\left.\begin{array}{ll} \textit{Uniaxial Tension} & \textit{Thin-Walled Vessel} \\ e_x = \exp(\epsilon_x) - 1 & e_c = \exp(\epsilon_c) - 1 \\ s_x = \sigma_x \exp(-\epsilon_x) & s_c = \sigma_c \exp(-2\epsilon_c) \end{array}\right\}. \tag{32}$$

Substitution of Eqs. 32 into 31 yields the final result

$$\left.\begin{array}{ll} \textit{Uniaxial Tension} & \textit{Thin-Walled Vessel} \\ e_x = \exp(n) - 1 & e_c = \exp(n/2) - 1 \\ s_{ult}/B = n^n \exp(-n) & s_{burst}/B = \dfrac{2}{\sqrt{3}}\left(\dfrac{n}{\sqrt{3}}\right)^n \exp(-n) \end{array}\right\}. \tag{33}$$

Of importance in Eqs. 33 are the conventional ultimate stresses in tension and in tube bursting. The dimensionless representation of these quantities (s_{ult}/B and s_{burst}/B) are plotted as a function of n in Fig. 5. As this figure shows, both s_{ult} and s_{burst} are strongly dependent

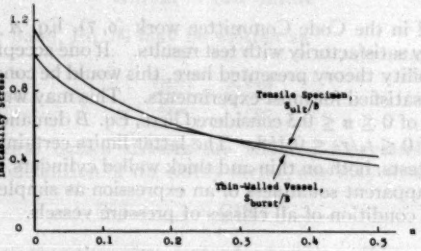

FIG. 5. The conventional instability stress.

on n, decreasing as n increases. Furthermore, at low values of n the tube stress at bursting is larger than the tensile stress at necking; at high n-values (material with great capacity for strain-hardening) the relationship is just the reverse.

The results obtained for conventional instability stresses in uniaxial tension and for pressurized cylinders can be readily compared. From Eqs. 33,

$$s_{ult}/s_{burst} = (\sqrt{3})^{n+1}/2.$$

Expanding the right side and ignoring terms of a higher order, there results

$$s_{ult} = \frac{\sqrt{3}}{2} s_{burst}\left(1 + \frac{n}{2}\ln 3\right)$$

which, by use of Eq. 10, modifies to

$$s_{ult} = \frac{\sqrt{3}}{2} p \frac{r_0}{t_0} (1 + 0.549n). \tag{33'}$$

The interesting aspect of this result is brought out when it is compared with current recommendations of the PVRC Subcommittee on Pressure Vessel Shell Theory for inclusion in the ASME Code. The result of intensive work in this field (6, 7) led to the proposal for a design equation in the following form:

$$S = \frac{\sqrt{3}}{2} p \left[\frac{r_0}{t_0} + 0.6 \right]. \tag{A}$$

In view of the fact that this proposal also assumes an equality of safety factors between tensile members and cylindrical vessels, Eq. A becomes directly comparable with Eq. 33'. This comparison reveals that the two results become equivalent when

$$t_0/r_0 = 0.916n. \tag{B}$$

As stated in the Code Committee work (6, 7), Eq. *A* was found to correlate very satisfactorily with test results. If one accepts the validity of the instability theory presented here, this would be conditioned upon Eq. *B* being satisfied for most experiments. This may well be the case; at the limits of $0 \leq n \leq 0.5$ considered here, Eq. *B* demands corresponding values of $0 \leq t_0/r_0 \leq 0.458$. The latter limits certainly account for all reported tests, both on thin and thick walled cylinders, and may thus explain the apparent soundness of an expression as simple as Eq. *A* for the bursting condition of all classes of pressure vessels.

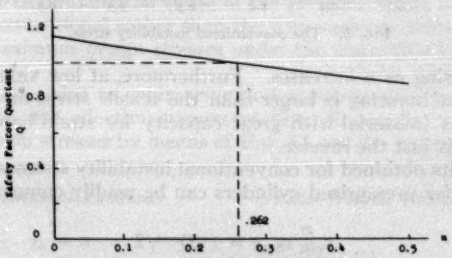

FIG. 6. Values of the safety factor quotient.

VI. SAFETY FACTORS FOR PRESSURE VESSELS

The next item of importance is the investigation of safety factors, when design of pressure vessels is based on the circumferential stress

alone. To this end one can define

$$Q = s_{burst}/s_{ult} \tag{34}$$

which, by Eqs. 33, becomes

$$Q = 1.155(0.577)^n. \tag{35}$$

This function is plotted in Fig. 6, and is seen to be almost linear. A good approximation to it can be obtained by expanding the exponential factor in series and ignoring all terms past the second one. This yields the approximate expression

$$Q \cong 1.155 - 0.634\,n. \tag{35'}$$

The significance of Eq. 35 is as follows. For materials subjected to uniaxial tension only, the design safety factor (as obtained from the tension test) represents a true safety factor. Let this be written in the form

$$s_{ult}/s_{des} = (\text{S.F.})_{tension}. \tag{36}$$

On the other hand, Eq. 34 may be written as

$$\frac{s_{burst}}{s_{des}} = Q\frac{s_{ult}}{s_{des}}.$$

If now identical design stresses are adopted for vessels and tensile members, by definition of the safety factor, one may write:

$$(\text{S.F.})_{vessel} = Q(\text{S.F.})_{tension}. \tag{37}$$

This means that the safety factor for bursting of thin-walled vessels depends upon the value of Q. As Fig. 6 shows, the answer is that this safety factor is larger than the tensile S.F. when n is less than 0.262. Above this value of n, the safety factor available for pressure vessels falls below the value inferred from tests on tensile specimens.

The important factor thus turns out to be n, the strain-hardening exponent of the metal. Ideally plastic materials ($n = 0$) exist only in text-books. In reality, most ductile metals in the annealed state have n values varying between 0.2–0.45, while cold-rolled metals have a range of about $0.05 < n < 0.25$, n being the lower the more severe the rolling operation. A few representative values of the strain-hardening exponent are shown in Table I; additional values, mostly on aluminum alloys, are given in (9). In general, nominal safety factors based on the tensile test will be on the conservative side for pressure vessels made of cold-rolled metals and either right or slightly unconservative for annealed materials.

In any case, the true safety factors for vessels cannot be far apart from those assumed on basis of the tensile test. Over the range of $0 \leq n \leq 0.5$, the deviation may run from an underestimate of 15.5 per cent to an overrating of 12.3 per cent, as shown in Fig. 6. For mild steels, in particular, n generally varies between 0.25–0.30. At these values, Q is almost exactly equal to 1.0. No wonder then that the different nature of safety factors due to geometry and stress system was not noted in bursting stress; small differences due to variations in the n-value of the particular steel composition and heat being tested, would have become obliterated through the normal scatter of experimental data.

VII. LIMITS OF STATIC COMBUSTION PRESSURES IN PIPES

The foregoing material permits a formulation of the limiting pressures that a thin-walled cylinder can withstand without rupture.

Suppose that the vessel operates at a design pressure, p_d, producing a circumferential engineering stress with a nominal safety factor (S.F.). The design stress, by analogy with Eq. 10, is given by

$$s_{des} = \frac{r_0}{t_0} p_d. \tag{38}$$

Using now Eqs. 37 and 35,

$$s_{des} = \frac{s_{burst}}{Q(\mathrm{SF})} = \frac{3^{(n+1)/2}}{2(\mathrm{SF})} s_{burst}. \tag{39}$$

Equating the above expressions,

$$p_d = \frac{3^{(n+1)/2}}{2(\mathrm{SF})} \frac{t_0}{r_0} s_{burst}. \tag{40}$$

Suppose now that the contents of the vessel undergo instantaneous combustion, during which the constant volume adiabatic pressure rises by a factor of μ. This is followed by a forced expansion of vessel size under isothermal conditions, as pictured in Fig. 7. The problem at hand is to determine the limiting value of μ, as influenced by the material properties and the selection of the (SF), that the vessel can sustain without rupture.

Several assumptions must be made at the outset. Apart from assuming that the combustion process is adiabatic, (no heat transfer during the post-combustion period), the design pressure will be considered to induce only elastic stresses. Thus design strains become negligible as compared to instability strains. The combustion itself will be regarded as occurring uniformly throughout the vessel. As such, it is not preceded nor does it propagate into a detonation wave. Inertia

effects are omitted and the potential of stress wave propagation in the vessel wall will be ignored.

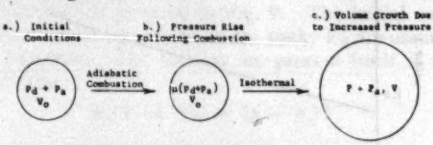

FIG. 7. Model of the combustion process.

With these assumptions, the perfect gas law stipulates that the product of volume and pressure must remain constant during the expansion process, so that

$$\mu V_0(p_d + p_a) = V(p + p_a) = K. \tag{41}$$

Considering now the definition of conventional strains, and combining this with Eqs. 33, there results

$$e_c = \frac{r}{r_0} - 1 = \exp(n/2) - 1$$

so that

$$r/r_0 = \exp(n/2). \tag{42}$$

Restricting the treatment to a unit length of pipe one may write

$$V/V_0 = (r/r_0)^2 = \exp(n) \tag{43}$$

by making use of Eq. 42. One can rewrite next Eq. 41 in the form

$$\mu = \frac{\dfrac{p}{p_d} + \dfrac{p_a}{p_d}}{1 + \dfrac{p_a}{p_d}} \frac{V}{V_0}$$

which, upon substitution of Eqs. 38, 40 and 43, becomes

$$\mu = \frac{\dfrac{2(SF)}{(\sqrt{3})^{n+1}} + \dfrac{p_a}{p_d}}{1 + \dfrac{p_a}{p_d}} \exp(n) \tag{44}$$

where the (SF) corresponds to the nominal engineering value valid for the tensile test. In most practical cases $p_d \gg p_a$, that is, $p_a/p_d \ll 1$. This permits simplification of the foregoing result to

$$\frac{\mu}{(SF)} = \frac{2 \exp(n)}{(\sqrt{3})^{n+1}}. \tag{45}$$

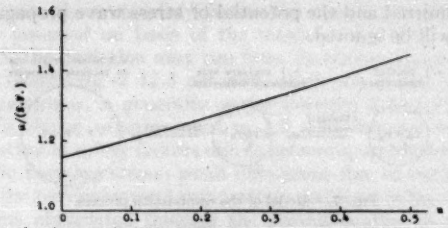

FIG. 8. Limiting values of the $\mu/(SF)$ ratio.

This dimensionless expression, plotted in Fig. 8, gives the maximum values of the factor by which the pressure rise at adiabatic combustion may exceed the conventional safety factor without provoking bursting of the vessel. This factor is seen to exceed 1.155 at all values of n.

Equation 45, as represented by Fig. 8, is almost linear. It can be very satisfactorily replaced by the simple expression

$$\frac{\mu}{(SF)} \cong 1.155 + 0.520\, n \qquad (45')$$

obtained from an infinite series expansion of Eq. 45 after ignoring terms beyond the first order. As Fig. 8 shows, the relative safety against destruction of the vessel increases with the strain-hardening exponent. This tends to favor annealed materials and those with a large strain-hardening ability; in particular, stainless steel, annealed copper and mild steel.

VIII. GAS VOLUME COMPRESSION AT BURSTING

With the results already in hand, one can proceed to investigate a diverting side-question: if the geometry and material properties of the

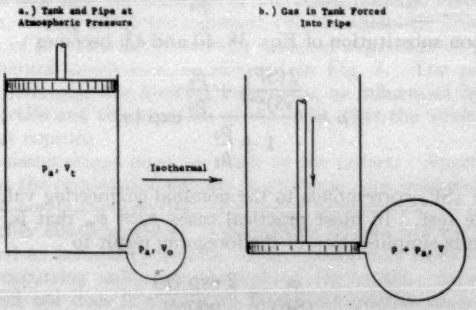

FIG. 9. Model of bursting process by gas compression.

circular cylinder are known, how much total volume of gas can be pumped into it prior to bursting.

The process itself is pictured in Fig. 9. The initial volume in the pipe, V_0, and the make-up volume in the tank, V_t, are assumed to be at atmospheric pressure. The blowing up process itself is taken to be isothermal, so that

$$p_a(V_t + V_0) = (p + p_a)V. \tag{46}$$

Therefore,

$$\frac{V_t}{V_0} = \left(\frac{p}{p_a} + 1\right)\frac{V}{V_0} - 1. \tag{47}$$

At bursting $p = s_{burst}t_0/r_0$, and by substitution of Eqs. 33 and 43 there results

$$\frac{V_t}{V_0} = \left[\frac{\frac{2}{\sqrt{3}}\left(\frac{n}{\sqrt{3}}\right)^n \exp(-n)B}{\frac{p_a r_0}{t_0}} + 1\right] \exp(n) - 1.$$

Letting $s_a = p_a r_0/t_0$ stand for the circumferential stress corresponding to an atmospheric pressure, the above expression simplifies to

$$\frac{V_t}{V_0} = \frac{2}{\sqrt{3}}\left(\frac{n}{\sqrt{3}}\right)^n \frac{B}{s_a} + \exp(n) - 1. \tag{48}$$

In view of the fact that the first term in Eq. 48 is generally of 0(3) while $\exp(n) - 1$ is of 0(0) for $0 < n < 0.5$, it is wholly permissible to omit the latter. There remains

$$\frac{V_t}{V_0} = \frac{2}{\sqrt{3}}\left(\frac{n}{\sqrt{3}}\right)^n \frac{B}{s_a}. \tag{49}$$

The curve for $2n^n/3^{(n+1)/2}$ is shown in Fig. 10; these values are seen

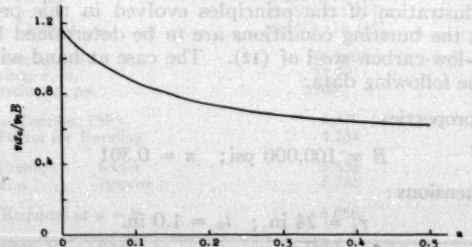

FIG. 10. Gas volume compression in bursting.

to vary from 1.155 to 0.620 as n goes from 0 to 0.5. The interesting part is that for the limiting combustion pressure examined before B did not enter the solution, and only n determined the available safety of margin, as exemplified by Eq. 45. Now, however, B appears in Eq. 49. Since the hardness factor, B, is apt to be a very large number, its influence on V_t will overshadow the contribution of the strain-hardening exponent, which is 2:1 at best as pictured in Fig. 10.

IX. EXPERIMENTAL RESULTS AND NUMERICAL EXAMPLE

Information on the true stress-strain curve of materials is scarce throughout the literature. This is unfortunate in view of the fact that this curve would shed much more light on the true behavior of materials than the conventional tensile test. The latter, performed by the thousands, is representative of no fundamental property of the material; its value stems only from the practical assessment that decades of engineering practice came to associate with it.

What little information on true stress-strain properties could be assembled at a hasty survey of the technical literature[6] is given in Table I.

TABLE I.—*True Stress-Strain Properties of Some Metals, for* $\sigma_x = B(\epsilon_x)^n$.

	Aluminum		Copper				Low Carbon Steel, Annealed	Stainless Steel, Annealed
Type of Metal	24S-T Clad	75S-O Bare	Hard-Rolled	5–7% Rolled	Annealed	Annealed		
Hardness Factor, B, in 1000's psi	100.0	54.5	60.0	65.5	65.5	60.0	100.0	190.0
Strain-Harding Exponent, n	0.168	0.145	0.066	0.303	0.375	0.375	0.301	0.415
Reference No.	10	12	11	13	13	11	12	12

This table lists the reference used as source material, and presents the two basic material properties, B and n. Hard-rolled metals are seen to have low values of n (between 0.066 and 0.168) while annealed materials exhibit higher rates of strain-hardening (n varying from 0.301 to 0.415). The hardness factor, B, is lowest for the softest metal, copper; aluminum has an intermediate B value while steels have the largest hardness factors.

As an illustration of the principles evolved in this paper, let us assume that the bursting conditions are to be determined for a vessel made of the low-carbon steel of (12). The case at hand will be represented by the following data:

Material properties:

$$B = 100{,}000 \text{ psi}; \quad n = 0.301$$

Vessel dimensions:

$$r_0 = 24 \text{ in.}; \quad t_0 = 1.0 \text{ in.}$$

[6] Reference 9, recently published, contains some added information on the strain-hardening exponent of materials.

Pressures and initial stresses:

$$p_a = 14.70 \text{ psi}; \quad p_d = 490 \text{ psi}$$
$$s_a = 352.8 \text{ psi}; \quad s_{da} = 11{,}760 \text{ psi}$$

Pressure rise at constant-volume adiabatic combustion:

$$\mu = 8$$

The material itself will be assumed to be SAE 285, Grade B steel, having an allowable working stress of 12,500 psi and a minimum tensile strength of 50,000 psi. The Code allowable pressure under these conditions is, assuming a joint efficiency $E = 0.95$ and an internal radius $R = 23.5$ in,

$$P = \frac{SEt}{R + 0.6t} = 492.7 \text{ psi.}$$

This value was used for setting the design pressure. The design stress was then calculated on basis of Eq. 38.

The following values can now be read off from the figures shown in this report:

$$\sigma_{eff}/B = 0.589; \quad \text{from Fig. 4}$$
$$s_{burst}/B = 0.504; \quad \text{from Fig. 5}$$
$$Q = 0.978; \quad \text{from Fig. 6}$$
$$\mu/(\text{SF}) = 1.321; \quad \text{from Fig. 8}$$
$$(V_i/V_0)/(B/\sigma_0) = 0.680; \quad \text{from Fig. 10}$$

TABLE II.—*Bursting Conditions for Vessels of Numerical Example.*

Item		Numerical Value	Reference Equation
s_{ult}, psi		51,500	33
Conv. Strain to Necking in Tension, e_z		0.351 (35.1%)	33
Stresses at Bursting	Effective, σ_{eff}, psi	58,900	30
	True Circumf., σ_{burst}, psi	68,000	31
	Conv. Circumf., s_{burst}, psi	50,400	33
Strains at Bursting	Effective, ϵ_{eff}	0.521 (52.1%)	30
	Log. Circumf., ϵ_c	0.1505 (15.05%)	31
	Conv. Circumf., e_c	0.1624 (16.24%)	33
Thickness at Bursting, t, in.		0.8603	14
Radius at Bursting, r, in.		27.90	14
Pressure at Bursting, p, psi		2100	10
Safety Factor in Tension, (SF)		4.379	36
Actual Safety Factor for Bursting		4.283	37
Allow. Adiab. Comb. Pressure Rise, μ	Exact	5.658	44
	Approx.	5.785	45
Nominal (SF) Required at $\mu = 8$		6.056	45
Gas Volume Compr. to Bursting, V_i/V_0	Exact	193.1	48
	Approx.	192.8	49

The latter value is also applicable to σ_{burst}/B.

With these data in hand, one can proceed to calculate the salient conditions at bursting and the safety factors inherent in the structure. This information is assembled in Table II.

By way of comments, it may be mentioned that the difference of the tensile safety factor, listed as 4.379 in Table II, from the generally assumed value of 4.0 is due in part to the joint efficiency factor, E, and in part to having used actual values rather than Code specifications in these calculations.

In most other aspects the results are self-explanatory. It is to be noted that, while the circumferential bursting strain (conventional = 16.2%) is much lower than the tensile necking strain of 35.1%, the engineering stresses at bursting and in tension are almost equal. Hence, safety factors do not differ much, as would be expected for a material with $n = 0.301$. Table II also shows that the conventional safety factor would have to be raised from 4.379 to 6.056, if the vessel were designed to withstand an 8-fold volume expansion in combustion. The unpressurized air volume needed to take the vessel to bursting turns out to be 193 times the initial volume of the vessel.

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ON THE RELATIVISTIC DOUBLETS OF SPECTRAL LINES

BY

R. H. PENFIELD [1] AND H. ZATZKIS [2]

ABSTRACT

The splitting of the energy levels predicted by the Dirac equation for a particle moving in a central field is determined. The relativistic effects consist of (1) a common displacement of the energy levels, and (2) a split into doublets for all states whose angular momentum is not zero. This first effect is, of course, difficult to access experimentally while the second is more readily verified by experiment. The calculations are based on a method which treats the relativistic effects as a perturbation and final expressions are given for the level splitting in the special case of the harmonic oscillator and the rigid rotator.

I. INTRODUCTION

The Dirac equations for a particle moving in a central force field predict a splitting into doublets for all states whose orbital angular momentum quantum number l is different from zero. This splitting is caused by the interaction of the spin of the particle with its orbital angular momentum. It is the purpose of this paper to calculate this effect for two special cases, the spherical harmonic oscillator and the linear rotator. It turns out that in general the relativistic effects consist of two parts: A common displacement of the energy levels which is experimentally difficult to measure and secondly a splitting into doublets which is accessible to experimental verification. The following calculations are based on a perturbation calculation and presuppose that the solution of the corresponding non-relativistic wave equation is known exactly.

For the sake of clarity, we shall describe the main features of the general theory, adhering closely to the presentation and notation given by Weizel (1).[3]

II. GENERAL FORMULATION

Dirac's four-component equations for a particle of rest mass m_0 can be reduced to two, 2-component wave functions Φ and belonging to the energy value $m_0c^2 + \epsilon$, where $\epsilon \ll m_0c^2$. If the potential energy V is a function of r ($r = \sqrt{x^2 + y^2 + z^2}$) only, the wave equation for Ψ reads

$$H^0\Psi + H'\Psi + H''\Psi - \epsilon\Psi = 0 \qquad (1)$$

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[3] The boldface numbers in parentheses refer to the references appended to this paper.

where

$$H^0 = -\frac{\hbar^2}{2m_0}\nabla^2 + V, \tag{2}$$

$$H' \cong -\frac{\hbar^2}{4m_0^2c^2}\frac{dV}{dr}\frac{\partial}{\partial r} - \frac{p^4}{8m_0^3c^2}, \tag{3}$$

$$H'' = \frac{\hbar^2}{4m_0^2c^2r}\frac{dV}{dr}(\tau\cdot\mathbf{L}). \tag{4}$$

τ is the vector spin operator whose components are the three Pauli spin matrices

$$\tau_x = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \tau_y = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_z = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tag{5}$$

and

$$\mathbf{L} = \frac{1}{i}[\mathbf{r}\times\nabla] = -i[\mathbf{r}\times\nabla]. \tag{6}$$

In the zeroth approximation the energy belonging to the quantum state k is

$$\epsilon_k^0 = \delta_k \tag{7}$$

with δ_k the eigenvalue of the Schrödinger equation

$$H^0\psi_k - \delta_k\psi_k = 0. \tag{8}$$

(k is short for the composite quantum number n, l, m.) The corresponding wave functions are

$$\Psi_\xi = \xi\psi_k \tag{9}$$

and

$$\Psi_\eta = \eta\psi_k. \tag{10}$$

ξ and η are two mutually orthogonal unit vectors in the reduced two-dimensional spin space. The operators $\mathbf{L}$ and τ commute with each other since they act on different variables.

In the first approximation the energy values ϵ_k are increased by the amounts

$$\Delta\epsilon_k = H_{kk}' + H_{kk}''. \tag{11}$$

Of course, it is to be kept in mind that the matrix elements H_{kk}' and H_{kk}'' are themselves 2-by-2 matrices because to each quantum state k belong two spin vectors ξ and η.

The diagonal elements of H_{kk}' are

$$H_{\xi\xi}' = H_{\eta\eta}' = -\frac{\hbar^2}{4m_0^2c^2}\int\int\int \psi_k^* \frac{dV}{dr}\frac{\partial\psi_k}{\partial r} r^2 dr d\Omega$$

$$-\frac{1}{8m_0^3c^2}\int\int\int \psi_k^* p^4 \psi_k r^2 dr d\Omega$$

$$d\Omega = \sin\theta d\theta d\varphi. \tag{12}$$

We have here again omitted the indices kk behind the matrix elements $H_{\xi\xi}'$ and $H_{\eta\eta}'$. The non-diagonal elements vanish, that is,

$$H_{\xi\eta}' = (\xi^*\cdot\eta)\int\int\int \psi_k^* H' \psi_k r^2 dr d\Omega \tag{13}$$

because η and ξ are mutually orthogonal. We note that the contribution of H_{kk}' to the energy is merely a shift of the level. It is the same for ξ- or a η- state; hence no splitting of the energy level results. In other words, any existing degeneracy will not be removed.

The situation is different in the case of the perturbation H''. The 2-by-2 matrix H_{kk}'' need not be diagonal, but we can always rotate the axes in such a manner that H_{kk}'' does become diagonal without destroying the diagonality of the H_{kk}'. If this has been done, we obtain the relations

$$(\tau\cdot\mathbf{L})\xi\psi_k = (\tau\cdot\mathbf{L})_{\xi\xi}\xi\psi_k \tag{14}$$

and

$$(\tau\cdot\mathbf{L})\eta\psi_k = (\tau\cdot\mathbf{L})_{\eta\eta}\eta\psi_k \tag{15}$$

where $(\tau\cdot\mathbf{L})_{\xi\xi}$ and $(\tau\cdot\mathbf{L})_{\eta\eta}$ are the diagonal elements of the diagonal matrix $(\tau\cdot\mathbf{L})$. In order to compute these elements, we start with the operator identity (2)

$$(\tau\cdot\mathbf{A})(\tau\cdot\mathbf{B}) = (\mathbf{A}\cdot\mathbf{B}) + i(\tau\cdot(\mathbf{A}\times\mathbf{B}))$$

and $\mathbf{A}$ and $\mathbf{B}$ are any two dynamical vector operators, which commute with the spin operator τ. If $\mathbf{A}$ and $\mathbf{B}$ are specialized to $\mathbf{L}$, we obtain

$$(\tau\cdot\mathbf{L})^2 = \mathbf{L}\cdot\mathbf{L} + i(\tau\cdot(\mathbf{L}\times\mathbf{L})); \tag{16}$$

but

$$\mathbf{L}\times\mathbf{L} = i\mathbf{L}, \tag{17}$$

and therefore

$$(\tau\cdot\mathbf{L})^2 = \mathbf{L}^2 - (\tau\cdot\mathbf{L}). \tag{18}$$

If we denote the operator $\tau\cdot\mathbf{L}$ by M, the equation

$$M^2 + M - L^2 = 0 \tag{19}$$

results, where

$$L^2\psi_k = -\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\psi_k) - \frac{1}{\sin^2\theta}\frac{\partial^2\psi_k}{\partial\varphi^2}. \tag{20}$$

Since ψ_k contains the spherical harmonic $P_l^m(\theta)e^{im\varphi}$ as a factor, we have

$$L^2\psi_k = l(l+1)\psi_k. \tag{21}$$

The matrix elements M_{ll} and $M_{\nu\nu}$ are therefore

$$\begin{aligned} M_{ll} &= l \\ M_{\nu\nu} &= -(l+1). \end{aligned} \tag{22}$$

Thus H'' yields the two diagonal elements

$$H_{ll}'' = \frac{\hbar^2 l}{4m_0^2c^2}\int\int\int \frac{\psi_k^*\psi_k}{r}\frac{dV}{dr}r^2dr d\Omega \tag{23}$$

and

$$H_{\nu\nu}'' = -\frac{\hbar^2(l+1)}{4m_0^2c^2}\int\int\int \frac{\psi_k^*\psi_k}{r}\frac{dV}{dr}r^2dr d\Omega. \tag{24}$$

The doublet splitting is therefore

$$\Delta\epsilon_k = H_{ll}'' - H_{\nu\nu}'' = \frac{\hbar^2(2l+1)}{4m_0^2c^2}\int\int\int \frac{\psi_k^*\psi_k}{r}\frac{dV}{dr}r^2dr d\Omega. \tag{25}$$

In contrast to the displacement $H_{ll}' = H_{\nu\nu}'$ the doublet splitting $\Delta\epsilon_k$ of the k^{th} quantum state is easily accessible to experimental verification. One exception to this splitting is the S state ($l = 0$) for which $\mathbf{L}\psi_k = 0$ and therefore also $H''\psi_k = 0$. Consequently, the S terms do not exhibit a splitting.

We will now apply Eq. 25 to the spherical harmonic oscillator and the rotational spectrum of the two-atomic molecule.

III. THE SPHERICAL HARMONIC OSCILLATOR

In this case $V = \frac{1}{2}br^2$ and Eq. 25 now becomes

$$\Delta\epsilon_k = \frac{(2l+1)\hbar^2 b}{4m_0^2c^2}\int\int\int \psi_k^*\psi_k r^2 dr d\Omega = \frac{(2l+1)\hbar^2 b}{4m_0^2c^2}. \tag{26}$$

The last equation follows from the fact that the ψ_k functions are normalized. Since

$$\frac{b}{m_0} = \omega_0^2 = \frac{4\pi^2c^2}{\lambda_0^2} \tag{27}$$

(ω_0 is the classical angular frequency of the oscillator), we can also write

$$\Delta\epsilon_k = \frac{(2l+1)h^2}{m_0\lambda_0^2}. \tag{28}$$

In order to express the splitting in terms of wave lengths $\Delta\lambda$, we can write

$$\Delta\lambda = \frac{hc}{\Delta\epsilon_l} = \frac{m_0 c\lambda_0^2}{h(2l+1)}, \quad l = 1,2,3, \ldots . \tag{29}$$

IV. THE DIATOMIC MOLECULE

We assume a potential of the form

$$V(\rho) = -2D\left(\frac{1}{\rho} - \frac{1}{2\rho^2}\right) \tag{30}$$

between the two atoms, where $\rho = r/a$ and a is the equilibrium distance between the two atoms (see Fig. 1). This potential function has been

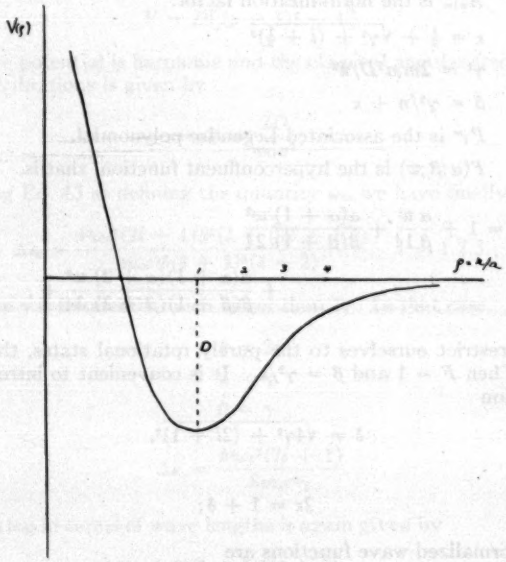

FIG. 1.

suggested first by Kratzer and Fues (3).

Let m_0 denote the reduced rest mass of the system, that is,

$$m_0 = \frac{m_0' m_0''}{m_0' + m_0''} \tag{31}$$

where m_0' and m_0'' are the rest masses of the two atoms. The Schrödinger equation for this system is

$$\Delta\psi + \frac{2m_0}{\hbar^2}\left(E + 2D\left[\frac{1}{\rho} - \frac{1}{2\rho^2}\right]\right)\psi = 0. \tag{32}$$

The normalized solution is

$$\psi = A_{nlm}\rho^{\kappa-1}e^{-\beta\rho}F(-n;2\kappa;2\beta\rho)P_l^m(\theta)e^{im\phi} \tag{33}$$

$n = 0,1,2, \ldots$

$l = 0,1,2, \ldots$

$m = -l, -l+1, \ldots, 0, \ldots, l-1, l.$

A_{nlm} is the normalization factor.

$\kappa = \frac{1}{2} + \sqrt{\gamma^2 + (l+\frac{1}{2})^2}$

$\gamma^2 = 2m_0a^2D/\hbar^2$

$\beta = \gamma^2/n + \kappa$

P_l^m is the associated Legendre polynomial.

$F(\alpha;\beta;w)$ is the hyperconfluent function, that is,

$$F(\alpha;\beta;w) = 1 + \frac{\alpha}{\beta}\frac{w}{1!} + \frac{\alpha(\alpha+1)}{\beta(\beta+1)}\frac{w^2}{2!} + \frac{\alpha(\alpha+1)(\alpha+2)}{\beta(\beta+1)(\beta+2)}\frac{w^3}{3!} + \ldots. \tag{34}$$

We shall restrict ourselves to the purely rotational states, that is, let $n = 0$. Then $F = 1$ and $\beta = \gamma^2/\kappa$. It is convenient to introduce the abbreviation

$$\delta = \sqrt{4\gamma^2 + (2l+1)^2}, \tag{35}$$

then

$$2\kappa = 1 + \delta; \tag{36}$$

and the normalized wave functions are

$$\psi_{0lm} = A_{0lm}\rho^{\frac{\delta-1}{2}}e^{-\beta\rho}P_l^m(\theta)e^{im\phi} \tag{37}$$

where

$$A_{0lm} = \left[\frac{(2l+1)(l-m)!(2\beta)^{\delta+2}}{4\pi a^3(l+m)!(\delta+1)(\delta+2)\Gamma(\delta)}\right]^{\frac{1}{2}} \tag{38}$$

and

$$\Delta\epsilon_k = \frac{2\hbar^2 D(2l+1)\beta^2(\delta-2\beta)}{m_0{}^2c^2a^2\delta(\delta+1)(\delta+2)}. \tag{39}$$

Since

$$\delta - 2\beta = \frac{\delta + (2l+1)^2}{1+\delta} \tag{40}$$

we can also write

$$\Delta\epsilon_k = \frac{2\hbar^2 D(2l+1)\beta^2(\delta+[2l+1]^2)}{m_0{}^2c^2a^2\delta(\delta+1)^2(\delta+2)}. \tag{41}$$

For small values of the displacement from the equilibrium position, the potential function can be written

$$V = D[(\rho-1)^2 - 1] \tag{42}$$

that is, the potential is harmonic and the classical angular frequency ω_0 for small vibrations is given by

$$\omega_0{}^2 = \frac{2D}{m_0a^2}; \tag{43}$$

thus, using Eq. 43 as defining the quantity ω_0, we have finally

$$\Delta\epsilon_k = \frac{\hbar^2\omega_0{}^2(2l+1)\beta^2(\delta+[2l+1]^2)}{m_0c^2\delta(\delta+1)^2(\delta+2)}, \quad l = 1,2,3. \tag{44}$$

In practice γ is frequently much larger than 1. In that case

$$\delta = 2\gamma$$
$$\kappa = \gamma$$
$$\beta = \gamma$$

and

$$\Delta\epsilon_k = \frac{\hbar^2\omega_0{}^2(2l+1)}{8m_0c^2\gamma}. \tag{45}$$

The splitting in terms of wave lengths is again given by

$$\Delta\lambda = \frac{hc}{\Delta\epsilon_k} = \frac{m_0c\lambda_0{}^2\delta(\delta+1)^2(\delta+2)}{h(2l+1)\beta^2(\delta+[2l+1]^2)}, \quad l = 1,2,3\ldots \tag{46}$$

and for large values of γ

$$\Delta\lambda = \frac{8m_0c\lambda_0{}^2\gamma}{h(2l+1)}, \quad l = 1,2,3,\ldots. \tag{41}$$

CONCLUDING REMARKS

We are planning to extend the method used here to hydrogen-like atoms and also to include higher quantum stages.

After finishing this paper, we found that a rigorous quantization of the relativistic hydrogen harmonic oscillator has been attempted by several others. Nikolsky (4) found that the wave functions remain finite at infinity. This is in contrast to Schrödinger's chemical solution and implies that the particle can even penetrate an infinitely high potential barrier. Sommerfeld and Welker (5) tried to avoid this difficulty by imposing artificial boundary conditions. Similar attempts have also been made by Auluck (6).

It seems to us that the difficulty lies in the formulation of a relativistically covariant potential energy. As Pauli (7) already pointed out, it contradicts the basic principles of relativity theory to use a potential energy to describe the effects of a force. This would imply that the effects of the force are transmitted with an infinite velocity. However, if the potential is constant in time (and this is the case here), it is still possible and even useful to retain the concept of a potential energy. In general, however, retardation effects ought to be taken into acount.

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ELASTIC MATERIALS UNDER AXIAL LOADING

BY

A. N. PROCTER[1]

Let mortals rejoice that such and so great an ornament of the human race has existed.
Newton's epitaph in Westminster Abbey.

INTRODUCTION

Sir Isaac Newton's method of research is defined with admirable clarity and brevity in the opening words of *Opticks* Book 1: "My Design in this Book is not to explain the Properties of Light by Hypothesis, but to propose and prove them by Reason and Experiments." The scientific method employed by Newton in *Opticks* (1)[2] and in *Principia* (2) can be employed to solve problems concerning the strength and elasticity of engineering materials. His discoveries in the realms of physics have provided the means of checking the results with the use of modern apparatus.

Newton's final conclusions on the subject of elasticity are contained in the third book of *Opticks*, Question 31 (3), and his suggestion that elasticity is produced by the elementary forces between particles of matter received fairly general assent during the two centuries following its publication. During the last half century the increased knowledge of the nature and reality of the internal forces in all kinds of materials has provided remarkable confirmation of Newton's intuition. He defined an elastic material in the following words: "If a Body is compact and bends or yields inward to Pression without any sliding of its Parts, it is hard and elastic, returning to its Figure with a Force rising from the mutual Attraction of its Parts." (1) Using these general principles as a starting point, a number of problems relating to the action of simple compressive and tensile forces on solid elastic materials may be reconsidered.

ELASTIC MATERIALS IN COMPRESSION

One of the simplest methods of investigating the elementary properties of elastic materials under axial loading is by compressing and crushing prisms or cylinders of brittle materials between the parallel plates of a powerful testing machine. Provided that the ends of the test pieces are machined to give an even bearing on the compression surfaces of the machine, it is inadvisable to interpose soft materials between the bearing surfaces and a thin layer of suitable lubricant will minimise the

[1] Nottingham, England.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

effect of the end friction. Since, however, the tests are to some extent a comparison of the properties of the materials and of the effect of the sizes and proportions of the test pieces, rather than an exact measurement of the specific strength in compression of the materials, it is important that the methods of testing should be uniform, in order to reduce, as far as possible, the number of variable factors to be considered.

Consider a uniform prism or cylinder of elastic material to which equal and opposite axial forces, F_A, are applied at the ends. At a normal cross-section, let it be assumed that the adjacent particles of the material exert equal and opposite unit forces or stresses, f_p, as in Fig. 1,

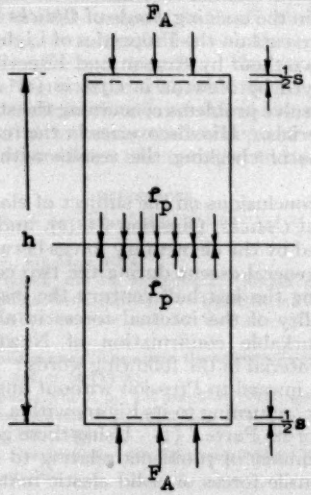


FIG. 1. Imaginary effect of axial particle forces on elastic material in compression.

so that the sum of the forces, f_p, is equal to the applied force, F_A. If, however, all the internal forces, f_p, are assumed uniformly distributed over the cross-section and also co-axial with the applied forces, F_A, there could be no strain normal to the axis of the test piece, since neither the applied nor the internal forces would have a component in that direction. This appears to follow directly from Newton's Second Law of Motion: "The rate of change of momentum is proportional to the applied force and takes place in the direction in which the force acts." Since it is established by experiment that under the conditions now considered there always is a transverse strain in an elastic material, it might

follow that the internal forces, f_p, are not all quite parallel with the applied forces. This being so, the forces, f_p, between the particles, may be resolved into unit axial forces or stresses, f_a, and unit shear forces or stresses, f_s, as indicated in Fig. 2, the transverse strain being produced by f_s, here described as the axial shear stress, the strain being radial, unlike shear strains produced by transversely applied forces.

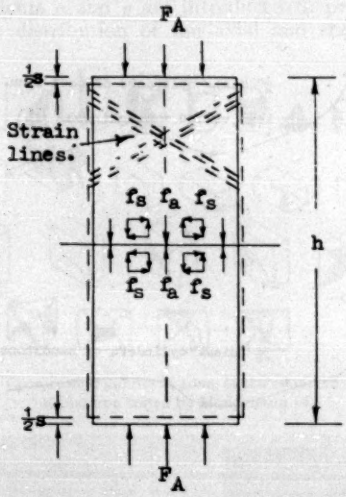


FIG. 2. Assumed stresses in elastic material in compression and actual strains.

With the exception of timber the compression failure of brittle materials extends throughout the height of the test pieces and tests have shown that the size and proportions of the test pieces have a considerable influence on the unit crushing strength, the principal factor in the strength of short test pieces being the ratio of the height to the thickness or diameter. A variety of cross-sectional shapes and sizes of test pieces have been used and for the purpose of simplification the results of tests on the crushing strengths of cylinders are illustrated in Fig. 3, where the values of the crushing strength are plotted against the ratios of height to diameter (h/d). The crushing strengths of the cylinders, $h/d = 0$ to 1.5 are progressively less as h/d increases. At certain ratios of h/d, about 2.0 to 1 for ½-in. diam. cast iron cylinders and about 1.5 to 1 for 1 to 5 mix concrete cylinders, the crushing strengths attain minimum values. Beyond this point the strengths are fairly constant, so that it appears that a change may have taken place in the internal stress con-

ditions. Similar results are obtained when prisms of other cross-section are used, so that it is clear that the phenomenon is caused mainly by the h/d ratio and by the properties of the materials in each case.

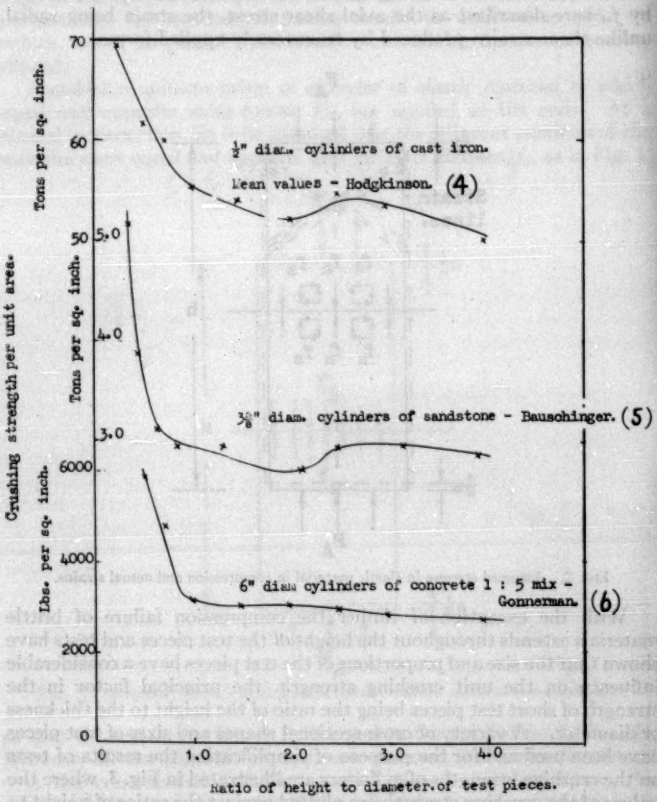


FIG. 3. The relation of the proportions of the test pieces to the crushing strength.

Assuming that the internal stresses are as illustrated in Fig. 1, it is difficult to explain these results, but if the arrangement of stresses indicated in Fig. 2 is assumed, it is possible to arrive at a simple mathematical representation of the facts by means of a strain energy calculation.

The internal energy per unit volume would be $\frac{1}{2}f_a\frac{s}{h}$, axial strain energy and $\frac{1}{2}f_s\phi$, shear strain energy, ϕ being the strain produced by f_s. Assuming the conditions of Hooke's Law, these might be written: $\frac{1}{2}E\frac{s^2}{h^2}$ and $\frac{1}{2}G\phi^2$ per unit volume. Since Hooke's Law may not be applicable, the qualifying coefficients α and η are introduced to provide for possible variations in the distribution of the axial and shear stresses. The

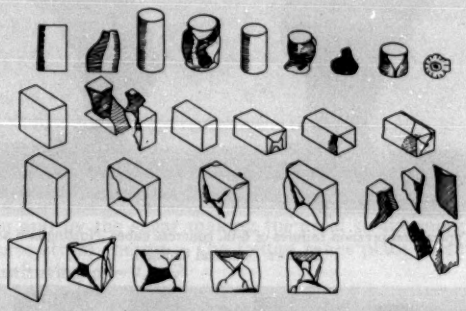


FIG. 4. Compression failures of some of the cylinders and prisms of cast iron tested by Hodgkinson (4).

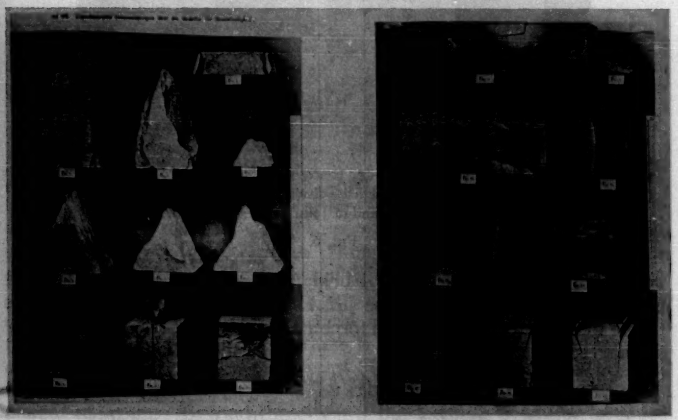


FIG. 5. Compression failures of sandstone cylinders and prisms tested by Bauschinger (5). Reproduced from illustrations by J. Albert, Munich.

Typical compression failures of 6-in. concrete cubes. Nottingham and District Technical College.

Typical compression failures of 6 × 12-in. concrete cylinders. Reproduced from Johnson's "Materials of Construction," p. 119, Eighth edition, 1946, by permission of M. O. Withey and James Aston, and of John Wiley & Sons, Inc.

FIG. 6.

volume of the test piece is Ah, A being the area of cross-section, so that the total strain energy would be $\frac{1}{2}\alpha EAs^2/h + \frac{1}{2}\eta GAh\phi^2$. Equating this to the work done by the applied force F_A, $\frac{1}{2}F_A s = \frac{1}{2}\alpha EAs^2/h + \frac{1}{2}\eta GAh\phi^2$ so that

$$F_A = \alpha EAs/h + \eta GA\phi^2 h/s. \qquad (1)$$

It appears from Eq. 1 that, when h is small, a large part of the applied force is absorbed in direct compression and only a small part in shear,

but for large values of h the applied force would be absorbed mainly in shear. Figures 4, 5 and 6 show the modes of failure in compression of cast iron, sandstone and of concrete test pieces and it can be seen that the short test pieces have failed mainly by crushing and the longer test pieces failed mainly by shearing, as suggested by Eq. 1. In order to find the condition when F_A reaches a minimum value, Eq. 1 should be differentiated with respect to h and equated to zero[3]:

$$\frac{dF_A}{dh} = -\alpha EAs/h^2 + \eta GA\phi^2/s = 0.$$

The critical value of $h = \frac{s}{\phi}\sqrt{\frac{\alpha E}{\eta G}}$ and the critical value of F_A is expressed as

$$\begin{aligned} F_A &= \phi A\sqrt{\alpha\eta EG} + \phi A\sqrt{\alpha\eta EG} \\ &= 2\phi A\sqrt{\alpha\eta EG}. \end{aligned} \tag{2}$$

From Eq. 2 it appears that F_A is now absorbed equally by the direct compression and by the shear making the axial strain energy equal to the transverse strain energy. The failure takes place in simple shear and by substituting $\phi = f_s/\eta G$,

$$F_A = 2\gamma A f_s\sqrt{\frac{E}{G}}. \tag{3}$$

In this equation $\gamma = \sqrt{\frac{\alpha}{\eta}}$ is a non-dimensional coefficient representing the effect of the shape of the section and certain properties of the materials and f_s is the critical shear stress.

ELASTIC MATERIALS IN TENSION

Since the axial elasticity of solids has a repulsive action under compression and an adhesive action under tension, it appears that there might be a neutral point between the two, where the Young's Modulus of an unstrained material would be zero. This was partially confirmed experimentally by D. K. Froman (7) on metal test pieces $\frac{3}{8}$ in. in diameter and $15\frac{3}{4}$ in. long. The values of E in tension were determined for a number of common metals under small loads. (See Fig. 7.) At less than 50 psi. the curves appear to rise from the direction of zero until they attain a maximum, where there is an abrupt change, similar, but opposite in direction, to the curves of Fig. 3 and having the nature of a relaxation that may be produced by the introduction of a second component of strain. Beyond these points the Young's Modulus becomes

[3] This method of calculation was devised by Newton in 1665 at the age of 23.

practically uniform, so that the constant elasticity of Hooke's Law might contain the two components identical with those that are here assumed to be present in the compression tests.

Since most elastic materials are stronger in shear than in tension, except in the case of timber, the failure under tension is usually a clear break normal to the axis and the possible presence of shear stress is not then apparent. Ductile materials like mild steel show Lüders' lines at about the yield point and the inclination of the lines suggests that the material may be subject to shear. Fine diagonal slip lines appear locally on the surface of the test pieces at or immediately beyond the proportional limit indicating that there is probably a partial shear failure

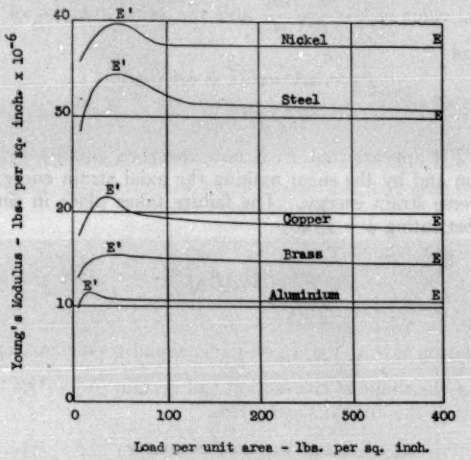


FIG. 7. Young's Modulus determined with small stresses (7).

and since this coincides with a deviation of the Young's Modulus from its constant value, it appears that the shear component of the axial elasticity has suffered merely a partial breakdown, because the original elasticity is restored when the load is removed or reduced.

There is some evidence that the thickness of the test pieces has an effect on the crushing strength of brittle materials and it is found that the tensile strength of elastic materials decreases with the increasing thickness or diameter of the test piece. Figure 8 shows the results of tests on hard drawn copper by Alkins (8), mean results of tensile tests on a number of kinds of cast iron by Campbell (9) and a series of tests on small diameter glass fibers by Griffiths (10). Tensile tests on other hard drawn metals and elastic materials give similar results.

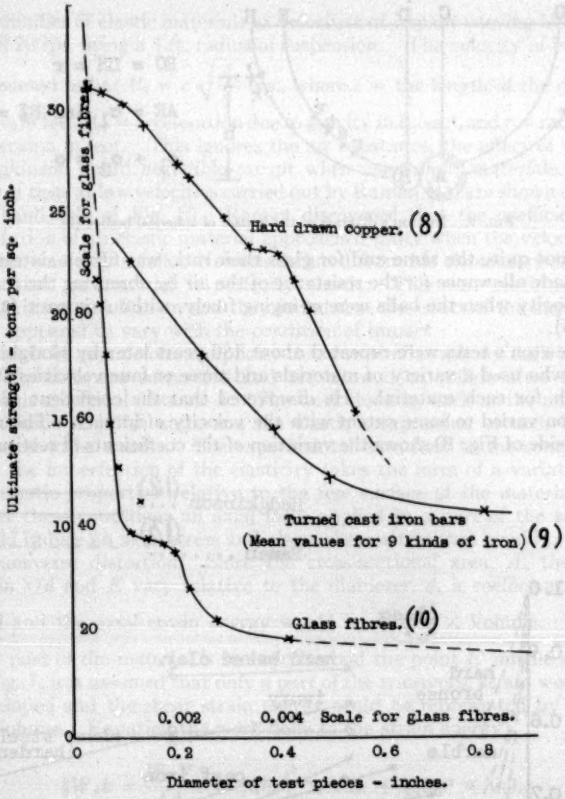


FIG. 8. The relation of the diameter to tensile strength of typical elastic materials.

THE IMPACT OF ELASTIC MATERIALS

Tests on the impact of spheres of various materials were made by Newton and also by some of his contemporaries (11). Two balls were suspended by threads and allowed to swing as shown in Fig. 9 that is a reproduction of Newton's own illustration in *Principia*. The materials he used were balls of wool, steel, cork and glass and he observed that the wool balls had a velocity of rebound about five-ninths of the velocity of impact, but for steel the velocities were nearly the same, for

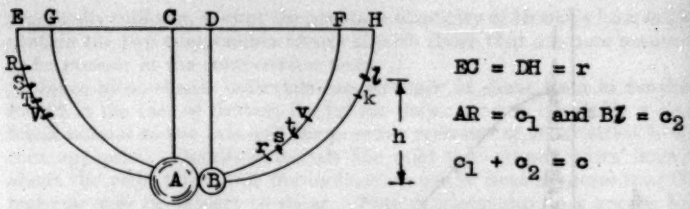


FIG. 9. Newton's diagram of the impact of spherical bodies (2).

cork not quite the same and for glass their ratio was fifteen-sixteenths. He made allowance for the resistance of the air by observing the change of velocity when the balls were swinging freely, without impact (2, pp. 21–25).

Newton's tests were repeated about 150 years later by Hodgkinson (12), who used a variety of materials and three or four velocities of approach for each material. He discovered that the coefficient of restitution varied to some extent with the velocity of impact. The right hand side of Fig. 10 shows the variation of the coefficients of restitution

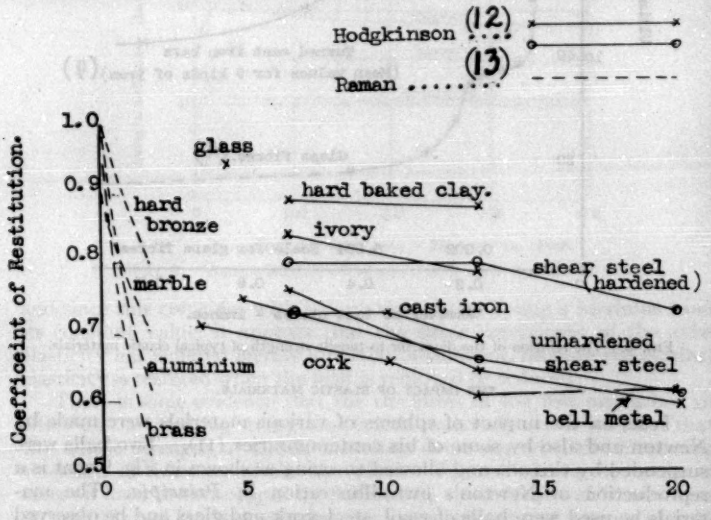


FIG. 10. The relation of coefficients of restitution to velocities of impact.

of a number of elastic materials at velocities of impact varying between 5 and 20 fps. using a 3-ft. radius of suspension. The velocity of impact is assumed to be: $V_1 = c\sqrt{\frac{A_g}{r}}$ fps., where c = the length of the chords $c_1 + c_2$ in feet, A_g = acceleration due to gravity in ft/sec^2, and r = radius of suspension in feet. This ignores the air resistance, the effect of which Hodgkinson found negligible except when using light materials. Results of tests at low velocities carried out by Raman (13) are shown on the left hand side of Fig. 10. Raman discovered that the coefficient of restitution of an elastic material approached unity when the velocity of impact was very small. Newton deduced from his elementary impact tests that no material is perfectly elastic. This was confirmed by Hodgkinson and Raman and they found that the effect of the imperfection appeared to vary with the condition of impact.

Let it be assumed for simplicity that two similar spheres each weighing W_A units are suspended and are allowed to collide after falling through a vertical distance h. The work done by each at the moment of collision is $\frac{1}{2}W_A h$ units and the elastic compression of each ball is assumed to be s units. The experimental data lead to the conclusion that the imperfection of the elasticity takes the form of a variation of the elastic properties[4] relative to the free surface of the material and under these conditions an axial force applied to a part of the surface would induce an axial stress and also a shear stress that would produce a transverse distortion. Since the cross-sectional area, A, the unit strain s/d and E vary relative to the diameter, d, a coefficient, α, is used and the axial strain energy would be $\frac{1}{2}\alpha E\frac{s^2}{d^2} \times$ Volume. When only part of the material is stressed beyond the point E' on the curves of Fig. 7, it is assumed that only a part of the transverse strain would be developed and the shear strain energy could be represented by $\frac{1}{2}\eta G\phi^2 \times$ Volume. Equating the work done to the strain energy:

$$\tfrac{1}{2}W_A h = \frac{W_A V_1^2}{4A_g} = \tfrac{1}{2}\alpha E\frac{s^2}{d^2} \times \text{Vol.} + \tfrac{1}{2}\eta G\phi^2 \times \text{Vol.}$$

The velocity of impact is

$$V_1 = \sqrt{\frac{2A_g}{W_A} \times \text{Vol.}\left(\alpha E\frac{s^2}{d^2} + \eta G\phi^2\right)}.$$

During the rebound the transverse distortion, being normal to the direction of impact, would not be reconverted into momentum and would

[4] Newton suggested that there was a gradual change in the properties of materials near the free surface (see letter to Robert Boyle dated February 28, 1679 (14)).

probably be converted into heat. Neglecting the energy lost by vibration, the velocity of rebound is

$$V_2 = \sqrt{\frac{2A_g}{W_A} \times \text{Vol.} \times \alpha E \frac{s^2}{d^2}}$$

and the coefficient of restitution is

$$\epsilon = \frac{V_2}{V_1} = \frac{\sqrt{\alpha E \frac{s^2}{d^2}}}{\sqrt{\alpha E \frac{s^2}{d^2} + \eta G \phi^2}}. \tag{4}$$

When the whole of the material is stressed beyond E' (Fig. 7), it is assumed that the whole of it is subject to shear stress, so that it would follow from Eq. 2 that the strain energy of compression and the strain energy of shear would be equal and Eq. 4 would then become

$$\epsilon = \frac{V_2}{V_1} = \frac{1}{\sqrt{2}} = 0.7,$$

but when none of the material is stressed beyond E' (Fig. 7) it is assumed that none of the material is stressed in shear and, neglecting losses, the velocities V_1 and V_2 would be equal, so that $\epsilon = 1.0$ very nearly. Thus for a material having elastic properties as assumed, the coefficient of restitution at stresses within the elastic limit would lie between 0.7 and 1.0, but when the maximum stress is beyond the elastic limit, leaving a permanent strain, the value would be reduced in proportion to the permanent/total strains.

The elasticity of the surface layers of some kinds of material appears to take the form of surface tension giving a high coefficient of restitution under normal conditions. This applies to amorphous materials such as thermo-plastics and glass which has a coefficient of restitution of about 0.94. Hard baked clay, green concrete, non-crystalline materials and colloids are in this category. Heat treatment is used to bring about the increase of the surface hardness of some metals or alloys that is usually associated with an increase in axial strength, although the elastic modulus is usually not appreciably altered. The heat treatment may, therefore, redistribute the elastic forces relative to the free surfaces so that the principal stresses are reduced, giving the material in effect an increase in strength.

Impact hardness is more easily investigated by means of the Shore Sceleroscope. Shore's own test results, mainly on metals, show that the Shore hardness is roughly proportional to the tensile strength, so that there appears to be an indirect connection between the two properties in

respect to metals, particularly steels. Turner (15) found that scleroscope hardness varies with the size and thickness of the material and he observed that the result of the test may vary with the nature of the support of the test piece. India-rubber gave a hardness number of 23—greater than mild steel—and soft pine wood 40—twice the value for grey cast iron—but teak, usually considered to be a harder wood than pine, gave a hardness number of 12. He found that the Shore hardness number of hard rolled copper was 30 and brass that would scratch it was 17. The hardness number of glass was 130—higher than hardened steel.

Newton considered hardness to be a necessary attribute of an elastic particle and more recently there have been theories identifying hardness with the molecular attractive forces and with the crystalline structure of metals and alloys (16). Impact hardness tests can not always be disassociated from the effects of ductility and plasticity, but a comparison of the hardness figures for similar kinds of materials may give some indication of their surface elasticity. It appears that the elasticity of a face-centered metallic crystalline material tends to radiate from the central axis toward the free surfaces, but the elasticity of an amorphous material like glass is directed from the free surfaces inwards. The orientation of the elastic forces might be reflected in the mode of failure of materials in tension and in compression and in the degree of variation of strength with the proportions and thickness of test pieces.

When a test piece is submitted to a gradually increasing axial load, the measured strains plotted against the intensity of loading may be used to indicate changes in the elastic conditions and to estimate the proportional limits of the materials that follow Hooke's law. In order to investigate the reasons why changes occur in the stress-strain relationship, the surfaces of a test piece may be inspected with a microscope or may be etched to reveal the development of fine cracks. Indirect methods of testing may be used to ascertain what is taking place in the interior; one way of doing this is by testing the effect of the increments of loading on the transmission of sound through the test pieces.

THE TRANSMISSION OF SOUND THROUGH ELASTIC MATERIALS

Propositions 47 and 48 of *Principia*, Book 2, contain a typically brilliant Newtonian research into the theory of sound, that forms the basis of seismic investigations and of modern ultrasonic research. He showed that the velocity of transmission of sound through an elastic medium or substance is proportional to the square root of the quotient of the elasticity divided by the density. Thus the axial velocity of sound through a rod or tube of solid matter would be:

$$V = \sqrt{\frac{E}{d}} \text{ cps} \qquad (5)$$

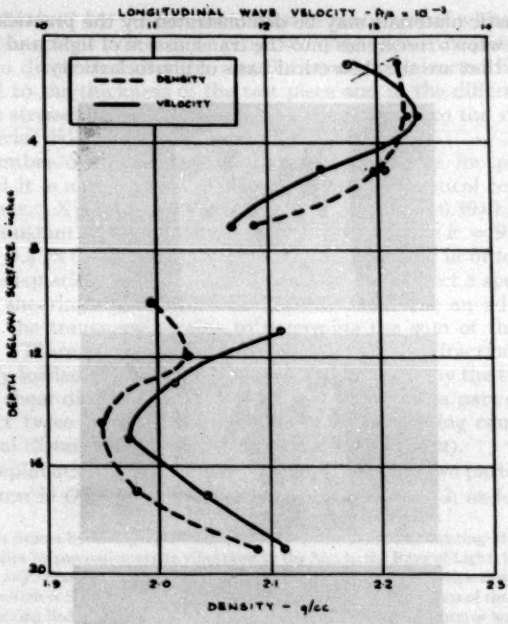
where E is the Young's Modulus in dynes per sq. cm. and the Density, σ, is in grams per cc. Newton's theory was tested experimentally by Wertheim and others and it was found that for rods of 15 different metals it gave results with an average error of $-\frac{1}{2}$ per cent and with maximum errors of about ± 7 per cent, a reasonably accurate confirmation (18). For rocks, timbers and other kinds of elastic materials there is a greater range of error, possibly because the materials are more variable, but the average of a number of tests has about the same accuracy as for metals. The formula does not appear to give good results for india-rubber and some other soft elastic substances. It is generally assumed for the purpose of seismic and geophysical experimental work that the velocity of sound in a large mass of elastic material would be a function of the bulk modulus, the modulus of rigidity and density (17), and in recent investigations (18) this has received satisfactory experimental confirmation. There is sufficient evidence available to show that the measured velocity of sound through a body may be used as a basis of comparison of the elastic properties at different depths or distances from the free surfaces.

Instruments have been developed to propagate a pulse of longitudinal ultrasonic vibrations between points on the opposite surfaces of concrete or other materials and to measure the times of transmission with an accuracy of about 1.0 per cent or better (19). It is found that when there is a discontinuity in the material, the velocity of transmission of the ultrasonic pulses is reduced, so that it is possible to detect the formation of cracks in a test piece under load by applying axial and transverse pulses. It has been shown that longitudinal cracks begin to form in the interior of 4-in. Portland cement concrete cubes at about a third of the crushing load and that in 12 × 6-in. diameter concrete cylinders the longitudinal cracks begin to form at about a half of the crushing load. When concrete is tested in tension there is little evidence of longitudinal cracks forming in test pieces, but there is evidence of transverse cracking shortly before failure.

The relationship of the wave velocity of ultrasonic vibrations to the density of a concrete road slab of 20 inches thickness is shown in Fig. 11 (19).[5] There was nearly a linear relationship and from this it appears by substitution in formula 5 that the elasticity of each part of the concrete was approximately proportional to the cube of the wave velocity.

When the material under compression is polished glass, the formation of the cracks inside the test pieces may be observed until the glass finally disintegrates into a cloud of small flying particles. Figure 12 shows glass cubes that were tested in compression and the pile of fragments that were collected after the final rupture. The cracks were evidently produced by the shear stresses indicated in Fig. 2 and the shear stresses appear to be caused by the variation of the elasticity of the materials adjacent to the free surfaces. The presence of shear strains in trans-

[5] Reproduced by permission of the Controller of H. M. Stationery Office.



The wave velocity was plotted against the density measured at various depths below the surface of the concrete and the results showed an approximately linear relation as indicated below.

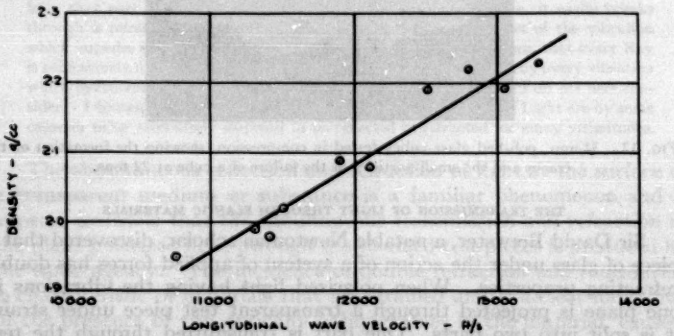


FIG. 11. The relation between the wave velocity and the density of concrete (19).

parent elastic materials may be demonstrated by the practical application of Newton's researches into the transmission of light and the origin of colours that are the theoretical basis of photoelasticity.

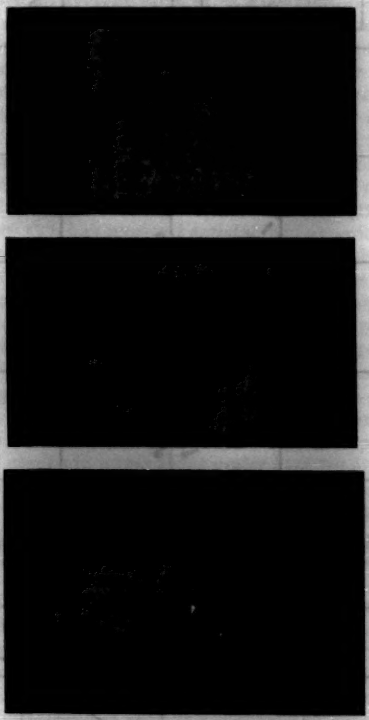


FIG. 12. 34-mm. polished glass cubes tested in compression, showing the formation of the cracks and the small particles at the failure of a cube at 22 tons.

THE TRANSMISSION OF LIGHT THROUGH ELASTIC MATERIALS

Sir David Brewster, a notable Newtonian scholar, discovered that a piece of glass under the action of a system of applied forces has double refracting properties. When polarized light having the vibrations in one plane is projected through a transparent test piece under strain, it is split into two parts. One part is transmitted through the test piece, but the other part is double refracted and is retarded. The inter-

ference obtained by the recombination of such rays of white light produces colour bands that may be projected onto a screen or photographed. Wertheim discovered that the retardation of the refracted light is proportional to the thickness of the test piece and to the difference of the principal stresses at a given point, but proportional to the strain when the material did not follow Hooke's Law (21).

A number of transparent materials are avilable for photoelastic work and it is necessary to determine their stress-optical constants by experiment. Xylonite ($E = 0.3 \times 10^6$ psi. and $\sigma = 0.39$) has a stress-optical constant about ten times greater than glass ($E = 9 \times 10^6$ psi. and $\sigma = 0.2$ to 0.27). Ditchburn (20) observes that in order to verify the stress equation completely, it is necessary to subject a specimen to a variable shearing force and it has been found to be an advantage to measure the transverse strains to determine the sum of the principal stresses. There is reason to believe that the double refraction caused by an axially loaded test specimen is probably produced by the two zones of opposed shear distortion indicated in Fig. 2, causing a part of the light to refract twice before emerging, the retardation being caused by the additional distance travelled by the refracted rays (22).

An explanation of the splitting of light rays into two parts was given by Newton in *Opticks*, Book 2, Part 3, Proposition 12, as follows:

> . . . as Stones by falling upon Water put the Water into an undulating Motion, and all Bodies by percussion excite vibrations in the Air; so the Rays of Light, by impinging on any refracting or reflecting Surface, excite vibrations in the refracting or reflecting Medium or Substance, and by exciting them agitate the solid parts of the refracting or reflecting Body, and by agitating them cause the Body to grow warm or hot; that the vibrations thus excited are propagated in the refracting or reflecting Medium or Substance, much after the manner that vibrations are propagated in the Air for causing Sound, and move faster than the Rays so as to overtake them; and that when any Ray is in that part of the vibration which conspires with its Motion, it easily breaks through a refracting Surface, but when it is in the contrary part of the vibration which impedes its Motion, it is easily reflected; and, by consequence, that every Ray is successively disposed to be easily reflected, or easily transmitted, by every vibration which overtakes it. But whether this Hypothesis be true or false I do not here consider. I content myself with the bare Discovery, that the Rays of Light are by some cause or other alternately disposed to be reflected or refracted for many vicissitudes.

The simultaneous reflection and refraction of light at the surface of a transparent medium or substance is a familiar phenomenon and it appears probable that the simultaneous transmission and refraction of the plane polarized light through the strained test pieces is produced by the same agency. The incidence of double refraction and retardation is characteristic of materials that are strained and is a clear indication of the presence of two optically oblique zones in the material, caused, possibly, by shear strain.

NOTATIONS USED IN THE PAPER

One dimension

c = length of chords
d = diameter
h = height
r = radius
s = total axial strain
x = a co-ordinate.

Two dimensions

A = cross-sectional area
E = Young's Modulus
f_a = axial stress
f_p = particle stress
f_s = shear stress
G = modulus of rigidity
V_1 = velocity of impact
V_2 = velocity of rebound

Three dimensions

A_g = acceleration of gravity.

Four dimensions

F_A = axial force
W_A = axial load

Two negative dimensions

Ω = density.

Non-dimensional

α = axial coefficient
γ = a coefficient
ϵ = coefficient of restitution
η = shear coefficient
σ = Poisson's ratio
ϕ = shear strain

NOTE: In order to conform to conventional notation, time is considered to be a negative dimension. Small letters represent one dimension and capital letters two dimensions. Thus: Elasticity; length^{-1} × mass × time^{-2} has plus two dimensions and density: length^{-3} × mass has minus two dimensions and is here represented by an inverted capital (23).

Newton with his prism and silent face,
The marble index of a mind for ever
Voyaging through strange seas of thought alone.
Wordsworth at Trinity College

Acknowledgment

The writer acknowledges with thanks the expert assistance of Miss Gilling in the preparation of this paper.

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Transparent Magnetic Oxides.—A new class of magnetic oxides, structurally distinct from ferrites, has recently been discovered. These materials, known as rare-earth-iron garnets, are transparent, permitting the internal magnetic domain structure to be seen with a polarizing microscope.

The discovery of ferrimagnetism in these garnets was made first at the Laboratoire Electrostatique et de Physique du Metal of the Institute Fourier in Grenoble, France, and independently by S. Geller and M. A. Gilleo of Bell Telephone Laboratories, where the optical and magnetic resonance behavior of the garnets is being studied by J. F. Dillon, Jr., employing single crystals grown by J. W. Nielsen.

Most magnetic materials, metals and ferrites alike, are opaque to visible light. Thus, their internal magnetic structure is not visible, and the way magnetic domains are oriented within them has been inferred from reflection of polarized light by their surfaces or by delineating domain boundaries at these surfaces with colloidal magnetic oxides.

Yttrium iron garnet, $Y_3Fe_2(FeO_4)_3$, is the most completely studied member of this new garnet family. It has a Curie temperature of 545° K. and a spontaneous magnetization at zero temperature and infinite field of 4.96 Bohr magnetons per molecule, close to the theoretical value of 5.0. This magnetization results from superexchange interactions through the O^{2-} ions between Fe^{3+} ions in crystallographically different positions in the cubic lattice. The number of these interactions per Fe^{3+} ion in yttrium-iron garnet is 3/5 that in a ferrite and, correspondingly, the observed Curie temperature of 545° K is 0.64 of that for magnetite (848° K).

Of great interest is the fact that yttrium-iron garnet contains magnetic ions with but a single valence. Both X-ray and neutron diffraction studies have shown clearly that, unlike the ferrites, the interactions between identical magnetic ions wholly occupying two different crystallographic sites are responsible for ferrimagnetism in the garnet structure.

In yttrium-iron garnet the width of the ferrimagnetic resonance absorption line at 9300 and 24,000 megacycles per second depends on crystallographic direction. Line widths of only 0.8 oersted were observed along the [100] direction in thin disks at 10,000 megacycles per second, when the temperature was held at 540° K. These line widths increased with decreasing temperature and passed through peak values of several hundred oersteds between 65° K. and 4.2° K.

A polarized light beam passing through the transparent magnetic domains in rare-earth-iron garnets has its plane of polarization rotated in one direction in one domain but in the opposite direction in an adjacent and oppositely magnetized domain. This Faraday rotation is wavelength-dependent and amounts to several degrees per mil of thickness; it makes the domains within the crystals clearly visible. The internal domain structure can therefore be studied over a wide range of temperatures and magnetic field conditions. Thus, for the first time, it is now possible to correlate Faraday rotation in a magnetic material with spectroscopic data over a broad temperature range.

THE FRANKLIN INSTITUTE

MINUTES OF THE ANNUAL MEETING

January 15, 1958

The Annual Meeting of The Franklin Institute was held at 8:15 P.M. in the Lecture Hall. Dr. S. Wyman Rolph, President, called the meeting to order with approximately 230 members and guests in attendance.

The President stated that the minutes of the November 20, 1957 meeting were published in the December issue of the JOURNAL. There being no corrections or additions the minutes were approved as published. The President announced that the minutes of the Stated Meeting of December 18, 1957, will be published in the January 1958 issue of the JOURNAL and will be submitted for approval at the next meeting.

The President called on Dr. George S. Crampton for the report of the Tellers of Election, appointed in accordance with the By-Laws to count the votes for the ten Managers to be elected. Dr. Crampton reported that 1,908 ballots were received, of which 1,829 were legal and 79 defective, and the nominees received votes as follows:

Henry B. Bryans	1,819	Morton Gibbons-Neff	1,819
Francis J. Chesterman	1,824	Gaylord P. Harnwell	1,816
James Creese	1,825	Harold S. LeDuc	1,818
Rupen Eksergian	1,822	W. Laurence LePage	1,815
W. H. Fulweiler	1,822	Emery W. Loomis	1,823

The President declared the above members were thereby elected Managers of The Institute for a term of three years, and thanked Dr. Crampton, Dr. Joseph Hepburn and Mr. Howard Stoertz for their services.

The President called upon the Secretary to read the report submitted by the Trustees of the Elliott Cresson Medal showing the financial transactions for the year, which, in accordance with the Deed of Gift which forms the basis of the Award, must be presented at the Annual Meeting of The Franklin Institute. The report follows:

ELLIOTT CRESSON MEDAL FUND

Report of the Trustees to The Franklin Institute

January 15, 1958

PRINCIPAL

Principal of the Fund (representing the original $1,000 plus transfers thereto from Unexpended Income and profit from disposition of investments)		$4,304.73
Invested as follows:		
$4,000 U. S. Treasury 2½% bonds, due 6/15/1962–67 at book value	$4,071.13	
Cash in Western Savings Fund Society	233.60	
	$4,304.73	

INCOME

Income of the Fund for 1957

Interest on bonds (less amortization)	$ 126.29	
Interest on bank balance	6.07	$ 132.36
Expenses		
Medals, certificates and cases		255.00
Expenses in excess of Income for the year 1957		$ (122.64)
Unexpended Income at January 1, 1957		2,359.65
Unexpended Income at December 31, 1957		$2,237.01

Invested as follows:

$2,000 U. S. Treasury 2½% bonds, due 6/15/1962-67 at book value	$2,035.75
Cash in Fidelity-Philadelphia Trust Co.	201.26
	$2,237.01

(s) *Henry B. Allen*
HENRY B. ALLEN

(s) *John Frazer*
JOHN FRAZER
Trustees

The President read a summary of the Annual Report of the Board of Managers for 1957. The Annual Report for 1957 will be mailed to the members of The Franklin Institute.

President Rolph announced that it was with considerable pleasure that he had the honor, of introducing the speaker of the evening, who would present the first of the William B. Coleman Lectures, which lectureship was established late in 1956. He then introduced Dr. James B Austin, Vice President for Research and Technology of the United States Steel Corporation who spoke on "Basic Research—Foundation of Steel's Progress Today."

Dr. Austin spoke of the broad research program of the United States Steel Corporation and the growth of research laboratories in private industry, stressing the need for basic as well as applied research. Dr. Austin commented on the importance of the performance of research work, particularly basic research, being done by private industry as well as universities and private laboratories such as The Franklin Institute. He emphasized the relative importance of basic and applied research.

The meeting was adjourned by the President at 9:30 P.M. with a rising and enthusiastic vote of thanks to Dr. Austin for his very fine presentation.

WILLIAM F. JACKSON, JR.
Secretary

MEMBERSHIP

The following elections were approved on January 9, 1958 by the Membership Committee by authority given to it by the Board of Managers.

ANNUAL

Sam Alewitz
Eleanor W. Ashkenaz
George M. Axilbund
Stanley Bernstein
Paul A. Blattler
Harold L. Bloom
John F. Byrom
John Castleberry, Jr.
Laurence R. Cederbaum
Frank J. Columbo
Anthony L. Consalvi
George D. Copeland
Alan Z. Corey
Woodrow Darling
C. A. Erkert
Bernard E. Featherman
Franklin T. Flaherty
Donald C. Fontaine
Kenneth H. Gordon
F. G. Gravalos
Robert H. Hilderbrand
Harry F. Irwin
Eleanor C. Knoke
Alfred Leith
Donald J. MacDonald
Bernard P-G MacGrann
Dennis McCoskrie
S. W. Melidosian
Daniel Miller
Frank A. Novicki
Charles Penrose, Jr.
Gladys R. Phillips
Joseph A. Porter
Robert C. Reed
C. H. Rice
Milton L. Rock
George A. Rosenbloom
Harry H. Rowley
Daniel R. Sabre
Ronald H. Silverman
Robert K. Smith
Allen W. Stewart
Louis F. Stone
Nellie Alice Stoner
Edwin L. Sutton
August J. Vella
Howard L. Volgenau
A. A. Wall
J. F. Walls
Cecil M. Waterbury
Daniel S. Whiteman, Jr.
Benton E. Williams
James W. Williams, III
Robert D. Woldow
Lewis Merrick Wood

EDUCATOR

W. H. Boghosian
Isabel M. Geesaman
Louis C. Green
Dale T. Harroun
William W. Hassler

NON-RESIDENT

Kenneth F. Thomas
Henry F. Zielinski

NECROLOGY

Nicholas Cascetti '43
B. T. Gordon '51
Loren C. Hurd '55
Herbert F. Leary '56
Edgar O. Oeters '39
Thornton Y. Olsen '16
Ben Snyder, III '46

THE FRANKLIN INSTITUTE LABORATORIES
FOR RESEARCH AND DEVELOPMENT
NICOL H. SMITH, DIRECTOR

AIR POLLUTION LAB REPORTS FROM LOS ANGELES

REPORT II *

BY

E. R. STEPHENS [1] AND R. C. DOERR [1]

The Franklin Institute mobile air pollution lab utilizing long-path infrared spectroscopy[2,3] for analysis of the gaseous constituents of the atmosphere has been operating in South Pasadena during the past year. The presence of ozone and peroxyacyl nitrite, a compound involved in ozone formation in the atmosphere during smog was previously reported from this lab.[4] With this field instrument most of the important pollutants in the Los Angeles atmosphere have been measured as a function of time and meteorological conditions. Smoggy air may be blown continuously through the cell in order to observe pollutants which might decompose on the cell walls, or it may be immobilized in the cell during an infrared scan if pollutant concentrations are changing rapidly. No drying agents or other absorbents are used.

On several days during the fall and early winter of 1956, chosen pollutants were monitored from before the morning traffic rush until well after the evening rush. Figures 1, 2 and 3 show the results of such measurements. The general shape of these plots can be interpreted in terms of meteorological factors, traffic patterns, and known photochemical reactions. First of all, the occurrence of significant amounts of any pollutant at ground level requires the formation of temperature inversion to prevent the normal ventilation of pollutants into the upper atmosphere. Two kinds of inversion commonly occur in the Los Angeles basin. First is the radiation inversion which develops during dark hours from the rapid cooling of the surface of the earth by radiation. This results in a layer of cooler air, just at ground level, in which pollutants emitted to the atmosphere can be trapped. Sunlight will, of course, destroy this inversion very quickly. The other type of inversion occurs at a somewhat higher level, one or two thousand feet above the

* This note is an excerpt from a paper presented at a Session on Air Pollution during the 22nd Midyear Meeting of the American Petroleum Institute's Division of Refining, in the Sheraton Hotel, Philadelphia, Pa., May 14, 1957.

[1] Chemical Kinetics and Spectroscopy Section, Chemistry & Physics Division.

[2] E. R. STEPHENS, P. L. HANST, R. C. DOERR AND W. E. SCOTT, "Reactions of Nitrogen Dioxide and Organic Compounds in Air," *Ind. Eng. Chem.*, Vol. 48, pp. 1498–1504 (1956).

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[4] JOUR. FRANKLIN INST., Vol. 263, p. 349 (1957).

ground, and arises from the adiabatic heating of a mass of air which descends from higher altitudes over the ocean. This is called a subsidence inversion and can occur during daylight hours. Pollutants emitted into the lower atmosphere can mix and be diluted by the air below the inversion level, but cannot break through this layer of warmer air to be dissipated into the upper atmosphere.

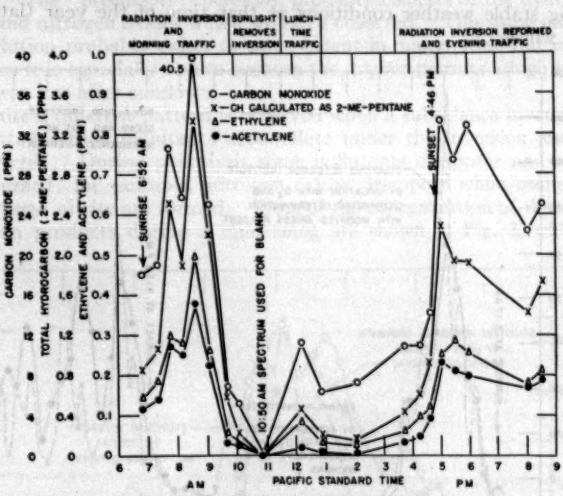

FIG. 1. Concentration of four pollutants at South Pasadena on December 17, 1956, a sparkling, smogless day.

The pollution pattern on a clear day is shown in Fig. 1. In order to obtain these data, a sample of outside air was trapped in the cell approximately every 25 minutes and analyzed for four pollutants. Total hydrocarbon was measured at 3.4 microns, carbon monoxide at 4.6 microns, ethylene at 10.52 microns, and acetylene at 13.7 microns. Presumably a radiation inversion existed at 6:50 A.M. when the first measurements were made. The rapid buildup to a peak at 8:40 A.M. is attributed to the increase in traffic at this hour. The extremely rapid drop in the concentrations of all four pollutants between 8:40 A.M. and 10:50 A.M. is accounted for by the destruction of the radiation inversion by sunlight. Concentrations of the four pollutants were so low at 10:50 A.M. that this spectrum was used for the blank in calculating the remainder of the data. This was done so that the blank would show nearly the same carbon dioxide and water-vapor content as the sample spectra. Only small peaks in the concentrations of all four pollutants

are apparent at noon and, in general, the concentrations during midday were low. The sharp rise at about sunset is attributed to the reformation of the radiation inversion and the increase in traffic density. Although the concentrations of these four pollutants were monitored throughout the day on only one occasion, portions of this curve were observed sufficiently often to warrant regarding it as a normal pattern during stable weather conditions at that time of the year (late fall, early winter).

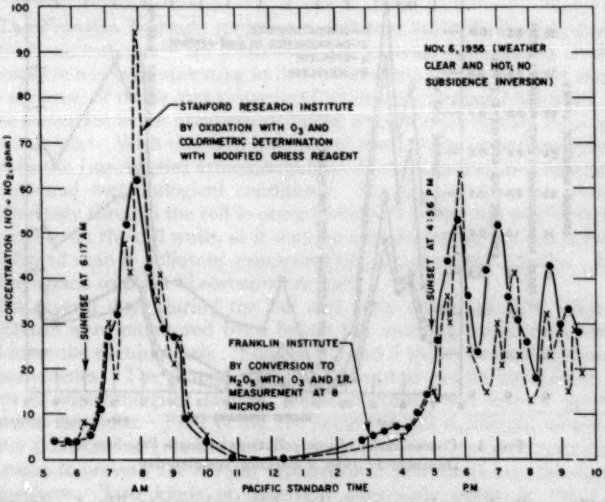
NOV. 6, 1956 (WEATHER CLEAR AND HOT; NO SUBSIDENCE INVERSION)
STANFORD RESEARCH INSTITUTE
BY OXIDATION WITH O_3 AND COLORIMETRIC DETERMINATION WITH MODIFIED GRIESS REAGENT
FRANKLIN INSTITUTE
BY CONVERSION TO N_2O_5 WITH O_3 AND I.R. MEASUREMENT AT 8 MICRONS
SUNRISE AT 6:17 A.M.
SUNSET AT 4:56 P.M.
CONCENTRATION (NO + NO_2, ppbm)
PACIFIC STANDARD TIME
A.M.
P.M.

FIG. 2. Concentration of nitrogen oxides at South Pasadena.

A similar pattern (Fig. 2) was observed for the total concentration of nitrogen oxides on November 6, 1956, under similar weather conditions; again peaks were observed at about 8:00 A.M. and in the evening, but none was found at midday. These data were obtained by the ozone-oxidation method previously described. Data obtained by Stanford Research Institute with the use of a colorimetric recorder based on modified Griess reagent are also shown in the figure.[8] The absence of the oxides at midday is explained by two factors. First, there was no inversion on the particular day in question, so that ventilation was good and pollution could not accumulate; and, second, any nitrogen dioxide in the air would be quickly photolyzed, and any nitric oxide

[8] SRI Technical Report No. 1, Report No. 6, SRI Project No. S-1793, "Instrument Development," issued Jan. 10, 1957 to the Air Pollution Foundation.

would soon be converted to peroxyacyl nitrite. The nitrogen oxides begin to accumulate surprisingly early in the afternoon, which suggests that the radiation inversion may form even before sunset.

This is probably a frequent or "normal" pattern in clear weather in the fall, winter, and spring and it matches closely the pattern of "precursor" concentration measured by Stanford Research Institute in 1954.[6] The "precursor," therefore, probably consists of the hydrocarbons and nitrogen oxides which we have observed. This same pattern of pollution probably exists to some extent in many cities, but in Los Angeles it is especially severe because the dry air permits a high radiative heat loss after sundown.

Quite a different pattern is observed when a subsidence inversion is present, for then pollutants accumulate under the inversion and are photolyzed. During photolysis some pollutants disappear and others are formed; for example, nitrogen oxides disappear while ozone and peroxyacyl nitrite are formed. Plots of the concentration of these two reaction products during a mild smog are shown in Fig. 3, which is

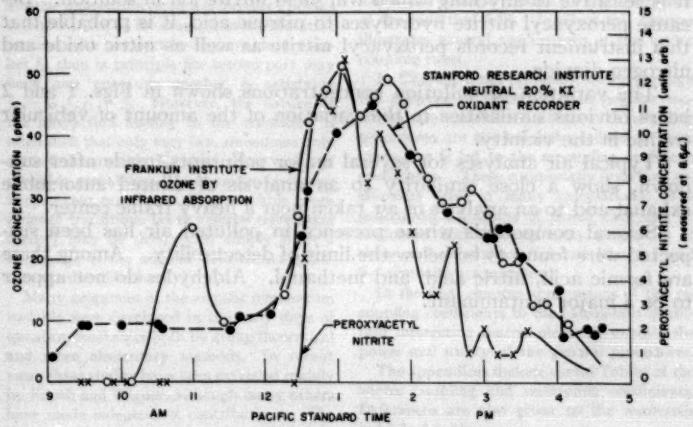

FIG. 3. Ozone and peroxyacetyl nitrite on November 12, 1956.

fairly typical of the pattern observed on a number of smoggy days. The peak concentrations of both occur at about one o'clock and diminish thereafter. A plot of the oxidant indicated by the 20 per cent potassium iodide recorder operated by Stanford Research Institute is shown for comparison in Fig. 3.[5]

[6] F. E. LITTMAN, H. W. FORD AND N. ENDOW, "Formation of Ozone in the Lower Atmosphere," *Ind. Eng. Chem.*, Vol. 48, pp. 1492–7 (1956).

It is noteworthy that the concentrations of all pollutants normally increase together when smog appears. This includes total hydrocarbons and carbon monoxide and shows that the rapid increases in concentrations of ozone and peroxyacyl nitrite do not represent photochemical *formation* of these compounds at the trailer location, but rather represent the *arrival* at the trailer location of a cloud of pollutant which has already undergone extensive photolysis. Typically, the hydrocarbon concentration may be approximately 1 ppm under these conditions (calculated as 2-methylpentane), and the carbon monoxide concentration may exceed 20 ppm. These concentrations are usually smaller than those observed during the morning and evening rush-hour peaks (Fig. 1).

An interesting correlation was made with the recorder used by Stanford Research Institute to monitor nitrogen oxides. Both recorder traces (nitrogen dioxide and nitric oxide plus nitrogen dioxide) showed a broad maximum, between 12:30 P.M. and 1:30 P.M., roughly corresponding to the infrared record of peroxyacetyl nitrite. Inasmuch as this recorder converts oxides of nitrogen to nitrite ion before measurement, it is sensitive to anything which will yield nitrite ion in solution. Because peroxyacyl nitrite hydrolyzes to nitrous acid, it is probable that this instrument records peroxyacyl nitrite as well as nitric oxide and nitrogen dioxide.

The variation of pollution concentrations shown in Figs. 1 and 2 bears obvious similarities to the variation of the amount of vehicular traffiic in the vicinity.

Typical air analyses for several major pollutants, made after sundown, show a close similarity to an analysis of diluted automobile exhaust and to an analysis of air taken near a heavy traffic center.

Several compounds whose presence in polluted air has been suspected were found to be below the limit of detectability. Among these are formic acid, nitric acid, and methanol. Aldehydes do not appear to be a major contaminant.

BOOK REVIEWS

ANGULAR MOMENTUM IN QUANTUM MECHANICS, by A. R. Edmonds. 146 pages, $6 \times 9\frac{1}{4}$ in. Princeton, Princeton University Press, 1957. Price, $3.75.

The conservation of angular momentum plays a more important role in the analysis of quantum mechanical systems than it does in classical systems. The fundamental reason for this is that the quantum mechanical angular momentum is quantized; it can assume only the values $(j^2 + j)^{1/2}\hbar$, where the quantum number j may have integer or half integer values. Another reason for the importance of this variable arises in the effect of selection rules in certain transitions. If a system with angular momentum quantum number j breaks into two parts, as for example in the radiative process, with one part having quantum number j_1, then in principle the second part may have any quantum number j_2 satisfying $j_1 - j_2 \leq j \leq j_1 + j_2$. However, the nature of the interaction leading to the transition is often such that only very few, sometimes only one, of the possible j_2 values arises with appreciable probability. In consequence it may happen that experimentally observable properties of transitions (as in angular correlation theory) may be determined simply from a knowledge of the angular momentum quantum numbers involved.

Many properties of the angular momentum variable were developed in the early days of quantum mechanics both by group theoretical and more elementary methods. In recent years these studies have been extended mainly by Racah and Wigner, although many others have made independent contributions. The monograph under review—*Angular Momentum in Quantum Mechanics*—gives a useful and unified account of the more recent developments.

The first four chapters of this work cover the elementary properties of angular momentum operators and states. The vector coupling coefficients, in terms of which the coupling of two angular momentum states may be achieved, are described in detail. In particular the symmetry properties of these coefficients, which are of importance in many applications, are derived. In the literature these coupling coefficients appear in many guises and in consequence, a comparison of different calculations is often somewhat confusing. A useful feature of this book is that the many different notations which are currently in use are noted and compared.

The remainder of the book deals with more recent work. Chapter 5 discusses the matrix elements of tensor operators, and in particular the dependence of the matrix elements on the magnetic quantum numbers; i.e., rotational dependence of the matrix elements. There is also a discussion of the vector spherical harmonics which are useful for the description of the radiation field. In effect this chapter illustrates general applications of the vector coupling rules.

In Chapter 6 the problems arising through the coupling of three and four angular momenta are considered. Since the coupling procedures are not uniquely defined in these cases, one is led to the recoupling or Racah coefficients. These rotationally invariant coefficients arise in many calculations. The procedures by means of which they can be evaluated and the symmetry properties of these and related coefficients (the 6-j and 9-j symbols) are examined.

In the final chapter applications of the recoupling coefficients to the calculation of certain interesting matrix elements exhibit the power and utility of the general procedures.

The appendices include useful Tables of the vector coupling and recoupling coefficients. References are also given to the numerous published tables.

Although the author makes no direct use of group theoretical methods, certain parts of the language of the book derive from group theory. A knowledge of the applications of group theory to quantum mechanics is not essential to an understanding of the book but certainly some familiarity with the subject would be helpful to the reader.

It is unfortunate, I think, that the author did not include the discussion of many more

applications to specific problems of interest. The book as it stands is a rather cold one and consists of the compact accumulation of many general results. For the motivation for this activity one is referred to the literature. The application of the general theory to representative problems would have made the book much more generally useful to the experimentalists and to students.

Despite this deficiency the book will be found useful as a reference by virtue of the precision of its definitions of the numerous coefficients involved and the clarity of its specification of their properties.

LEONARD EISENBUD
Bartol Research Foundation

NOTES ON ANALOG-DIGITAL CONVERSION TECHNIQUES, edited by Alfred K. Susskind. 410 pages, diagrams, 6 × 9 in. Cambridge (Mass.), The Technology Press, 1957. Price, $10.00 (paper).

Analog to digital converters (coders) and digital to analog converters (decoders) had to be developed before digital computers, characterized by great speed and flexibility could be linked to physical systems. This link may be either at the end of the physical systems, (for data processing), or at their beginning (for numerical machine tool control for instance), or inside them (for closed loop operation). Although all existing converters have been developed in the last decade, there were enough techniques investigated to justify the holding by MIT, in 1956 and 1957 of Special Summer Programs on Analog-Digital Conversion Techniques. This book is a revision and extension of notes prepared for these Programs by five staff members of the Servomechanisms Laboratory.

Sampling (referring to the input intermittently) and quantizing (approximating an analog quantity by a number) are so important in the estimation of the performance of analog-digital systems that a chapter of eighty pages, or one-fifth of the book, is devoted to the presentation of the significant results achieved to date in sampling and quantizing theory. The next two chapters, in which digital codes (especially reflected and redundant codes) and basic digital circuits are discussed, are mainly an introduction to readers not familiar with digital techniques. The following two chapters explain the principles of techniques actually used in the conversion, when the analog quantity is either a voltage or a mechanical displacement (translation or rotation). The last two chapters give an example of the design of analog to digital conversion equipment for a specific digital system, a Digital Flight Test Instrumentation (DFTI). The requirements placed on the conversion equipment by the specifications of the over-all system are discussed, and two coders developed to meet these requirements are explained in some detail. One coder is the optical shift position type which measures the rotational position of a shaft; the other is a digital camera type which measures the angular position of an object in an optical field. Both coders and the DFTI system have been developed by MIT.

The book, as the Program itself, is intended for engineers who may have been away for some time from academic work, hence usually only elementary mathematics are used and step by step derivation of equations shown. The results of the sampling theory cannot be obtained in that manner, and consequently are brought up in an intuitive manner, although they could be substantiated by a rigorous mathematical treatment; this chapter is quite outstanding, and its study is highly recommended to anyone interested in data processing. In all the chapters, the presentation is very clear and illustrated by a large number of sketches, graphs and circuit diagrams, which make the reading very easy.

All through the book, emphasis has been placed on principles of techniques, used or possible, and the determination of the accuracy which can be attained with each one. An attempt has been made to cover all the basic techniques used to date (the bibliography at the end of each chapter is quite extensive and references to very recent publications made), but no commercially available device is described. However, the experimental equipment developed by MIT is explained in detail, as an example of actual realization, and also because it shows applications of novel techniques. Of particular interest is the digital camera, which represents an attempt to eliminate the time consuming processing of photographic records, since digital information is obtained directly, but many improvements are necessary to increase the accuracy, and suppress the necessity of

mounting a light source on the object to be observed.

In a field as new as the one concerning the use of digital computers with physical systems, it is expected that theories and techniques may evolve rapidly. Many investigations are being carried out on the sampling theory, the digital scanning and processing of pictorial information and the use of the computer as intrinsic element of the converter, but as far as it can be seen presently, these techniques appear to be a natural extension of some techniques presented in this book (applied to the digital camera or the digital control of machine tool for instance). Consequently, this book is likely to provide an excellent introduction to the subject for the uninitiated reader since no previous background is assumed, and is a good source of fundamental techniques for the practicing engineer. In the last respect, an index would be an added convenience although the table of contents is quite detailed.

MARCEL A. MARTIN
General Electric Company

ELEKTRONENRÖHREN, by M. J. O. Strutt. 391 pages, diagrams, 6½ × 9¼ in. Berlin, Springer-Verlag, 1957. Price, DM 58.50.

This book is the third volume of a textbook of "Radio Communication Technique" edited by Nicolai Vonkorshenewsky, München, and Wilhelm T. Runge, Ulm am der Donau.

As the title indicates, this volume is devoted to the subject of electron tubes, their fundamental functioning, application and characteristic data. The topic "Electron Tubes" is used as a generic term. It includes transistors and crystal diodes besides the high vacuum and gas tubes with multi-grid systems for rectification, power amplification and gating, and television picture tubes as well as ikonoscopes. The constructionally quite different deci- and cm-wave tubes are omitted.

The subject is presented in three main sections: (A) Physics and basic engineering principles; (B) Electron optics based on electromagnetic fields; and (C) Data and properties of electron tubes presently in use.

In Section A the properties of the electron are discussed first with respect to its charge, mass, interaction with the atom and its electromagnetic wave nature. Second, the role of the electron in various materials is treated, such as electron motion in metals, semi-conductors, thermo- and field emission of electrons, the photo effect and secondary effects. This is followed by a presentation of the electrical and magnetic fields, both fundamentally and as they pertain to this subject. Finally, Section A closes with a chapter on the various materials and methods used in tube construction.

Section B is devoted to the control of the electron by means of electromagnetic fields. It starts with the diodes and their characteristics, followed by multigrid tubes, breaking the circuitry by the four-terminal-network theory. A whole chapter is devoted to the various problems of electron optics, and another to the spontaneous fluctuations in electron current.

In Section C the tubes are treated with respect to their applications, such as their use in the input stage, specialty tubes as gas tubes, phototubes, crystal diodes and transistors, application of tubes in output stages and finally, the various cathode-ray tubes.

The author's 21 years of industrial experience reflects in the presentation of the subject matter.

KARL SITTEL
The Franklin Institute Laboratories

ANALYTICAL DESIGN OF LINEAR FEEDBACK CONTROLS, by G. C. Newton, Jr., L. A. Gould, and J. F. Kaiser. 419 pages, drawings, 6 × 9 in. New York, John Wiley & Sons, Inc., 1957. Price, $12.00.

Feedback control is at least as old as 2000 B.C. for recorded laws of that time relate to these principles in connection with land irrigation in Babylonia. However, as late as World War I relatively crude control methods sufficed, and only in the last two decades have the exacting designs of servomechanisms been experienced. Even now, the authors of the present volume point out, the field of nonlinear systems analysis has advanced but little compared to linear systems. This book is concerned exclusively with the analysis of linear models. Non-linearity is considered only in connection with saturation signals.

George Newton and his co-authors have in this book tackled the fundamental ideas which are important to advanced analysis of servomechanisms. For example, it is recognized

that the complexity of most real devices is so great that an approximate mathematical model must be used for a satisfactory analysis. This is not easy, for in practice the engineer is confronted with numerous fixed elements, those which he has no part in selecting.

The maturity of this book shows in the care which is taken to distinguish analytical design from trial-and-error methods. To some extent this is the whole basis for the book.

Reaching far beyond the trial-and-error procedure, which is the method usually employed in first-course treatments of feedback control theory, the analytical design procedure employs a so-called performance index. Thus, in a single number a quality measure of system performance is derived. This analytical design method follows an approach to the design of feedback control systems started by Wiener at M.I.T. in the early 1940's.

The beginning chapters face squarely some of the necessary conceptual techniques required for this advanced servomechanism course, for instance, methods for expressing stochastic signals and probability-density functions. The major portions of the text are devoted to specific mathematical procedures such as the determination of correlation functions, the use of constraints, the treatment of unstable fixed elements, and examples illustrating the analytical method.

The excellence of this advanced book presenting effectively the state of the art of analytical design of linear feedback controls is a consequence of the brilliance of George Newton, who is Associate Director of MIT's Servomechanisms Laboratory, and his coauthors working at the same institution.

C. W. HARGENS
The Franklin Institute Laboratories

WORK SAMPLING, by Ralph M. Barnes. 282 pages, illustrations, 6 × 9 in. New York, John Wiley & Sons, Inc., 1957. Price, $7.95.

In 1934, L. H. C. Tippett published a report describing a statistical method of sampling a working population to measure operator and machine idleness. In 1950, these methods were introduced to American Industry. Since then the literature has not lacked for reports of their application.

In *Work Sampling*, Dr. Barnes has not added to this literature, but, rather, he has critically evaluated and summarized it. Briefly, his book may be described as a summary of the basic statistical theory of work sampling, a compendium of its various applications, and an efficient "cookbook" for those who wish to apply these methods. The real meat of this work is contained in the first one hundred pages and in Tippett's original report which is reproduced as the final chapter. A thorough knowledge of the material contained in these sections will certainly equip the reader to perform most types of work sampling studies. The practicing engineer will also appreciate the many nomographs, tables, and control chart techniques which eliminate the need for calculations, and for many this alone is worth the price of the volume. The rest of the book is merely exposition of the applications of the methods garnered from the literature. In this section the articles on performance sampling and on the use of punched card techniques should be of interest to all.

In summary it can be said that Dr. Barnes has produced a good introduction to the study of work sampling, and for those who wish a deeper knowledge he has thoughtfully included an ample bibliography.

FRANCIS X. MCLAUGHLIN
The Franklin Institute Laboratories

MATHEMATICS FOR SCIENCE AND ENGINEERING, by Philip L. Alger. 360 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1957. Price, $5.50.

Few books on mathematics are both instructive and readable at the same time. Popular works are usually so diluted as to be worthless to a reader seeking help with a real problem, while textbook writers always seem to be writing for someone who already knows all about it anyway. Mr. Alger's book, in refreshing contrast, is a fine example of mathematics written for learning and use.

The author covers a great deal of ground, from arithmetic to differential equations, with many side trips, without ever appearing superficial. He does this by concentrating on essentials and emphasizing the unity and simplicity of the mathematical ideas involved. His firm and clear manner of presentation makes for both sound learning and interesting

reading. He uses plain language always and physical illustration whenever possible, and his style resembles the sort of patient oral explanation characteristic of the best personal teaching.

By no means a cookbook, and completely unsuited to drill-type instruction (it has few problems, and no answers), this work should nevertheless be of great help to anyone seeking real insight into the physical applications of mathematics in engineering. Any engineer, from the beginning student to the practicing professional, can read this book with profit and pleasure.

WILLIAM C. YAGER
The Franklin Institute Laboratories

DYNAMIC PROGRAMMING, by Richard Bellman. 342 pages, 6 × 9¼ in. Princeton, Princeton University Press, 1957. Price, $6.75.

The author has here compiled and connected many of his results which have previously appeared in a scattered number of research journals. The central theme of the book is a method of solution of a single type problem which is capable of various statements.

We are given a physical system which changes state with time; we have some control over this change and seek to maximize a given payoff function. This process is called a decision process, and if a sequence of decisions is to be made it becomes a multistage decision process. This is the central problem of dynamic programming.

However, the term "dynamic programming" also includes the method of solution which is to find the optimal decision for any state of the system—not just those states occurring in the sequence. A sequence of decisions is called a policy. An optimal policy is an optimal sequence of decisions. The method now makes use of the author's Principle of Optimality: *An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.* The application of this principle leads to a functional equation which an optimal policy must satisfy. The author derives this equation for a great variety of problems.

In his preface the author states his case for dynamic programming. Toward this end he makes "a brief excursion into the philosophy of mathematical models." This short essay is in itself an excellent summary of the responsibilities of the scientist in choosing a mathematical model to fit his physical problem. So far as decision processes are concerned the author "unhesitatingly" chooses his dynamic programming.

The first two chapters are devoted to examples of an allocation process and a stochastic dynamic programming problem. In a sense it is beneficial to start with examples; however, the reader is apt not to realize the scope of dynamic programming from these two chapters.

In the third chapter the author states the theme of the book in a greater generality, which it deserves. He points out that an even greater generality is possible (in this volume the variables are finite dimensional vectors) and promises a second volume.

The fourth chapter is devoted to existence and uniqueness theorems for solutions of some classes of problems. These theorems rest upon a fundamental lemma which compares two optimal policies as functions of the state of the system. There is a misprint in the proof of this lemma which is most easily rectified by reversing the signs of the inequalities in (5), page 118. Otherwise the proof stands.

In the remaining seven chapters dynamic programming is applied to a variety of problems—some contemporary, some classical—with several results. For example, the solution to a class of calculus of variations problems is given as an envelope of tangents rather than the dual locus of points. In the theory of games another criterion is derived for playing non-zero sum, two-person games.

A large number of exercises and research problems are included at the end of each chapter. The reader is thus able to determine just how well he is following the thread of the argument.

The author constantly focuses his attention on the computational problems involved and often outlines methods suitable to digital computers. References to the promised second volume dealing with problems of greater difficulty and greater generality are scattered throughout this first volume. Some of these are rather tantalizing and the reviewer hopes

that the author will be encouraged to write another work.

SAMUEL G. KNEALE
General Electric Company

NEUTRON CROSS SECTIONS, by Donald J. Hughes. 182 pages, diagrams, 5½ × 8½ in. New York, Pergamon Press, Inc., 1957. Price, $5.00.

This book of 182 pages deals with all phases of the interaction of neutrons with matter. Neutrons, slow-intermediate-fast, falling in all regions of energy, are considered. Experimental results are copiously illustrated in figures showing curves which demonstrate the principles discussed in the text. It appears that this volume has been written with the idea of giving the inexperienced reader a "feeling" for neutron physics. Because of its brevity, the presentation is somewhat vague in spots, nuclear models, etc., being difficult to treat in a short discussion. There is some question as to the organization of the material; e.g., the last chapter, dealing with thermal neutrons, might have been given an earlier position in the text. There are one or two inaccuracies in the statement of physical principles which might be confusing to the unsophisticated student. However, this book may have a place not only as a quick summary for physicists desiring to become rapidly acquainted with the methods of neutron physics but also as collateral reading for students of nuclear physics at the advanced undergraduate or early graduate level.

C. E. MANDEVILLE
Bartol Research Foundation

POLYETHYLENE, by Theodore C. Kresser. 217 pages, illustrations, 5 × 7½ in. New York, Reinhold Publishing Corp., 1957. Price, $4.95.

The author has, in a pocket sized edition, adequately covered the various aspects of polyethylene: its history, general properties, basic chemistry, manufacture, processing, and applications. Though the text is directed primarily to persons who are interested in the practical uses of polyethylene, the text also contains information for the plastics engineer who is interested in knowing the technology of a material, particularly in the chapter on basic chemistry.

In showing the versatility and many applications of polyethylene, the author has correlated performance characteristics with application. For instance, polyethylene is imminently suitable for packaging fresh produce because it is impervious to moisture and yet passes oxygen and carbon dioxide, and is successfully used as underground tubing since it is so chemically resistant, inert to fungus, and mechanically tough over a range of temperatures. Moreover, he has shown how certain limitations of this material can often be easily overcome by lamination with another material, e.g., Kraft paper to improve tensile strength, cellophane to increase resistance to oils.

That polyethylene has tremendous potential in its present state evidently is appreciated by the author, but he has also pointed out that the technology is constantly being improved to yield superior products.

MARVIN PACKER
The Franklin Institute Laboratories

MYSTERIES OF SCIENCE, by John Rowland. 214 pages, 5½ × 8 in. New York, Philosophical Library, 1957. Price, $6.00.

Subtitled "A Study of the Limitations of the Scientific Method," this book is one of many being published today citing the limitations of scientific investigations. All good scientists know this, but it does no harm to be reminded of it once in a while. In fact, the historical treatment of the change in scientific philosophy from the Victorian era to the present day is perhaps the best feature of the work.

In the latter portion of his book, the author makes a strong plea for tolerance of some revolutionary ideas such as J. B. Rhine's Extra-Sensory Perception Theory, and calls for a thorough and rigorous investigation of such controversial topics as "water dowsing," "flying saucers," and interpretation of dreams. Regrettably, he spends too much time attacking pure science for not doing some spectacular work in these fields. He may have a point in urging research, however. The intelligent layman will find this an interesting and thought-provoking volume.

ROBERT W. NEATHERY
The Franklin Institute Museum

CHEMISTRY OF NATURAL AND SYNTHETIC RUBBERS, by Harry L. Fisher. 208 pages, illustrations, 5¼ in. × 9 in. New York, Reinhold Publishing Corp., 1957. Price, $6.50.

Rubber was used by the natives of the Amazon centuries before the first satisfactory vulcanizates were obtained by Charles Goodyear. It has not been until fairly recent times, however, that the structure of rubber was clarified and the mechanism of vulcanization is still being studied. Ironically, man produced and cured synthetic rubbers before unravelling the mystery of natural rubber. The chemistry of the synthetic polymers thus served to shed more light on the natural product's structure.

To the rubber chemist, used to recipes and table after table of moduli and elongations, this book is a refreshing change. The possible mechanisms of vulcanization, acceleration, and aging are discussed in the terms of the organic chemist, thus systematizing the grab bag of compounds which make rubber the most versatile elastomer.

After adequate discussion of the curing processes for rubbers, the author takes up the individual polymers ranging from the natural hevea rubbers to the recently introduced chlorinated polyethylenes ("Hypalon") and polytrifluorethylenes ("Kel F" elastomer). The history, production, and uses of these materials are discussed.

While the points considered in this book are kept brief, the main stream of rubber technology is well covered. For the reader desiring more detailed discussions, a bibliography is provided at the end of each chapter. The book is a good review of a field which has progressed mostly by trial and error and the author emphasizes the know why of rubber chemistry rather than the already voluminous know how.

R. H. HOLLINGER
The Franklin Institute Laboratories

THE PLANET VENUS, by Patrick Moore. 132 pages, plates, 5½ × 8½ in. New York, The Macmillan Co., 1957. Price, $3.00.

"Venus is a planet of mystery. Though nearer to us than any heavenly body apart from the Moon, it is the world about which we know least, and it guards its secret well." Thus begins the last paragraph of *The Planet Venus*.

Yet the author has written a book of 114 pages on this planet of mystery—on this planet of which we know nothing. Despite this, the author is well qualified to write this book—perhaps more so than any astronomer in the world today. Patrick Moore is a ranking astronomical observer who has observed with both small and large telescopes in England, France and this country. He is Secretary of the Lunar Section and Director of the Mercury and Venus Section of the British Astronomical Association. His observational skill of more than two decades permits the personal touch to be woven into his writings of a planet which he feels has been neglected too long.

The first three chapters of the book deal with historical facts, Venus as a world and the movement of the planet. These are all topics about which there is no mystery. These are simply known facts about the planet. The author then speaks of the elusive, evanescent markings found in the atmosphere of the planet and mention is made of the polar caps of this body and a strong case is made for them—but we don't even know whether the axis of rotation is in the proper position to give us polar caps. The axis of Venus has only to be lying on its side to make this possible. Polar caps must be treated with considerable reserve.

The rotation of Venus is covered and it is as much a mystery today as it was a century ago.

The atmosphere of the planet is discussed in detail in a superbly written chapter. His explanation for "The Ashen Light" is comprehensive. The author's revelation of Venusian (this reviewer will not use Cytherean as an adjective) aurora to explain this phenomenon is novel. Transits and Occultations, the Surface of Venus and finally Life on Venus close the book. Some appendices are included with significant data on the planet.

The book is superbly written with a wealth of anecdotal material and contains all the known material on the planet. It should serve as a reference book for a long time to come.

I. M. LEVITT
The Fels Planetarium of The Franklin Institute

INSTALLING ELECTRONIC DATA PROCESSING SYSTEMS, by Richard G. Canning. 193 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1957. Price, $6.00.

The problems connected with the installation of an EDP system are never fully anticipated and those which are, are often underestimated. A handy source of information on these problems and their practical solution would then be most beneficial.

This book indeed fills the need. The author has presented an interesting, clear and factual book aimed at management-level personnel and emphasizing the multitude and the magnitude of problems involving such items as time, cost and personnel.

The material for the book was acquired from actual case studies of companies using EDP systems in varied fields. The author has molded together the findings of all these companies for ease of discussion into one company whose history is presented. The time period involved is two and one-half years beginning with the placing of the order for the EDP machines. Another book by the same author entitled "Electronic Data Processing for Business and Industry" deals with events leading up to this period.

Some of the topics discussed in the period before actual installation of the machines are: organizational changes brought about by EDP; selection and training of personnel; general programming procedures; and programming the computer runs. In the installation period itself, the reader's attention is directed to the variables connected with the location of the computer. In the final period of converting from the present system to the EDP system, details of converting the files, testing the system, the reliability, accuracy and cost of operation are discussed. An excellent appendix is included which presents the human side of electronics and the characteristics of EDP personnel.

This book presupposes only a general familiarity with the basic concepts of EDP and may be highly recommended to those contemplating the installation of an EDP system.

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PUBLICATIONS RECEIVED

The following books have been received in the JOURNAL office for review. Lack of space prevents publishing more than this listing.

FINDING FOSSIL MAN, by Robin Place. 126 pages, plates, diagrams, 5¾ × 8½ in. New York, Philosophical Library, 1957. Price, $7.50.

ELEMENTS OF PHYSICS, by Alpheus W. Smith and John N. Cooper. Sixth edition, 671 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1957. Price, $7.50.

H. M. STANLEY: UNPUBLISHED LETTERS, by Albert Maurice. English edition, 185 pages, plates, 6 × 9¼ in. New York, Philosophical Library, 1957. Price, $7.50.

RELIGION, PHILOSOPHY AND SCIENCE, by Burnham P. Beckwith. 241 pages, 5½ × 8½ in. New York, Philosophical Library, 1957. Price, $3.75.

FUNDAMENTALS OF HYDRO- & AEROMECHANICS, by L. Prandtl and O. G. Tietjens, translated by L. Rosenhead. Unabridged and unaltered reproduction of the first edition, 270 pages, diagrams, 5½ × 8 in. New York, Dover Publications, Inc., 1957. Price, $1.85 (paper).

APPLIED HYDRO- & AEROMECHANICS, by L. Prandtl and O. G. Tietjens, translated by J. P. Den Hartog. Unabridged and unaltered reproduction of the first edition, 311 pages, diagrams, 5½ × 8 in. New York, Dover Publications, Inc., 1957. Price, $1.85 (paper).

BOOK NOTES

IDEAS, INVENTIONS AND PATENTS, by Robert A. Buckles. 270 pages, diagrams, 5½ × 9 in. New York, John Wiley & Sons, Inc., 1957. Price, $5.95.

In this concise text, the author's aim is to clarify the complicated patent laws. He discusses the principles that underlie patents in all fields of technology—mechanical, electrical, chemical, and nuclear. Carefully avoiding legal "jargon," the reader is taken step-by-step through a complete case history of an invention, thereby providing an unparalleled guide for those having no previous experience with the workings of the Patent Office and its attorneys. A valuable feature of the volume is a Glossary of common terms, and supplemented throughout are graphic illustrations from patents and patent drawings. Inventors and the technically trained will find this work a definite aid in acquiring a better understanding of patent matters.

TECHNICAL REPORT WRITING, by James W. Souther. 70 pages, 10¾ × 9¼ in. New York, John Wiley & Sons, Inc., 1957. Price, $2.95.

A compact treatise detailing the fundamentals and simplification of technical writing. This paper-bound manual clearly emphasizes the method and processes to be followed in the solution of reporting problems. The author's treatment is basically a design approach, involving the principles of engineering design, translated to the terms of writing.

SOVIET EDUCATION FOR SCIENCE AND TECHNOLOGY, by Alexander G. Korol. 153 pages, 6 × 9 in. New York, John Wiley & Sons, Inc. and Technology Press of MIT, 1957. Price, $8.50.

The material presented in this volume is a generalization of the entire education system of the Soviet Union. By examining its structure, its strengths and weaknesses and its potentialities, the author seeks to convey his views of Soviet education in the context of its goals, contrasting with those of the American educational system. The text describes the organization and evaluates the effectiveness of Soviet formal training in science and technology, reflecting the influence of Communism. The book is a useful contribution to the heretofore scant knowledge of Soviet education, as it aims toward a better understanding of its social and political implications.

PIONEERING IN INDUSTRIAL RESEARCH, by Kendall Birr. 204 pages, 6 × 9 in. Washington, D. C., Public Affairs Press, 1957. Price, $4.50.

The theme of this book is the growth and development of the General Electric Research Laboratory, its policies and problems, and an appraisal of its accomplishments. Particular attention is given to research in the fields of lighting, X-rays, electronics, chemicals, and metallurgy. Businessmen and scientists will be especially interested in the discussion of laboratory procedures and administration.

MATHEMATICS AND WAVE MECHANICS, by R. H. Atkin. 348 pages, diagrams, 5½ × 8½ in. New York, John Wiley & Sons, Inc., 1957. Price, $6.00.

A concise summary of the mathematics necessary for an intelligent understanding of quantum mechanics, beginning at about the intermediate calculus level and arranged to form an integrated course of study. The style is terse and formal, after the British manner, but explanations are quite clear. This book should be useful as a reference or as a text for a course in mathematical physics.

ELEMENTARY PRACTICAL ORGANIC CHEMISTRY, PART II, QUALITATIVE ORGANIC ANALYSIS, by Arthur I. Vogel, 320 pages, 5½ × 8½ in. New York, Longmans, Green & Co., 1957. Price, $4.50.

This text features an unusually comprehensive list of derivatives, and an excellent review

of reactions of selected classes of organic compounds. The series promises to be a valuable teaching tool in the field of organic chemistry.

NEW FRONTIERS OF KNOWLEDGE, a symposium by distinguished writers, scholars, and public figures. 125 pages, 6 × 9 in. Washington, D. C., Public Affairs Press, 1957. Price, $2.75.

Based upon a symposium held by the U. S. Information Agency, this volume presents a series of talks by distinguished writers, statesmen, and world-renowned figures. These talks were broadcast throughout the world by the Voice of America. The wisdom and vision of some of the most prolific minds of this age are included in this book, whose target is a better understanding of the hopes of humanity.

REASON AND CHANCE IN SCIENTIFIC DISCOVERY, by R. Taton, 171 pages, plates, 5¼ × 9 in. New York, Philosophical Library, 1957. Price, $10.00.

In this volume the author delves with penetrating insight, into the workings of the inventive mind. Examining the purpose and chance in the processes of scientific discovery, the text, not theorizing, prefers to illustrate by practical examples drawn from the works of distinguished scientists such as Poincare, de Broglie, Bernard, Galileo, Roentgen, Becquerel, the Curies, Leibnitz, Newton, Fleming, et al. Of unusual interest are the 32 illustrations, line drawings and reproductions of historic documents. The book undoubtedly would be a valuable addition to any scientist's library, and of particular interest to research workers.

IN SEARCH OF MAN, by Andre Missenard, 346 pages, diagrams, 6 × 9 in., New York, Hawthorn Books Inc., 1957. Price, $5.95.

An English translation from the French by Lawrence G. Blochman, this text attempts to answer a question: if man's development is governed by three major forces, chemical, physical and psychical, each being controlled by techniques with the range of human knowledge, how and when should these techniques be applied? Thinking readers will welcome this intense book, with its clarity of presentation, as a document of the first importance.

PRINCIPLES OF ELECTRICAL MEASUREMENTS, by H. Buckingham and E. M. Price, 600 pages, diagrams, 5½ × 8½ in. New York, Philosophical Library, 1957. Price, $15.00.

This text devoted to electronic measurement techniques, provides a new approach to some long-established problems. Included are valve methods, resonance and heterodyne measurements, and a fairly extensive treatment of the cathode ray oscillograph. A portion of the contents also deals with indicating instruments and deflectional methods which still play an important part in measurement practices, as well as sections on potentiometer and bridge methods, instrument transformers, integrating meters, measurements related to power systems, and the measurement of some non-electrical quantities. Covering the requirements of final degree and A.M.I.E.E. examinations, this volume is sufficiently comprehensive to be a very useful reference source to engineers and others requiring information on electrical measurements.

OLD STONE AGE, by Stevan Celebonovic and Geoffrey Grigson. 96 pages, illustrations, 9¼ × 12½ in. New York, Philosophical Library, no date. Price, $10.00.

An archaeological study in which the camera is advantageously used to illustrate, by a modern visual technique, the age-old interplay of man and nature. Second in the Nature and Art series, the reader's glance ranges over the relics of a million years of art and artifact, and enables him to observe the growth of man's consciousness of the significance of the human figure. Interwoven with 72 beautiful photographs, is an imaginative text by Geoffrey Grigson, whose aim is to interpret what might have passed in the minds of stone-age men by reason of these pictorial evidences. A volume worthwhile acquiring for the serious student of archaeology and anthropology.

CURRENT TOPICS

Corrosion Indicator Developed for Air Force.—A simple and inexpensive indicator which reveals the tell-tale signs of corrosion long before it affects Air Force equipment has been developed by the Air Research and Development Command. It is based on the principle that a thin metal film deposited on a suitable base changes in appearance when it corrodes. During exposure the electrical resistance of such film increases, since metal oxide is normally a poorer electrical conductor than the base metal.

Engineers who deveolped the process at ARDC's Wright Air Development Center, Ohio, deposited metal films of iron and other metals uniformly over glass and plastic bases through vacuum deposition—a process which "evaporates" metals in a high-vacuum chamber.

The metal is heated in a vacuum chamber until it evaporates. The condensation of the metal vapors deposits a film on the base material. This process is also used for decorative and protective coatings on such items as toys, automobile parts, and jewelery.

Inspection of stored Air Force equipment is usually accomplished at regular intervals. This requires opening the package, examining and represerving the contents, and then repackaging.

Present corrosion indicators used in packaging are either chemical or electrical devices. Their indicating properties are affected by high temperatures and cannot be used with some of the newer methods of Air Force packaging.

The metal film sensing elements can be used either visually or electrically to indicate a dangerous corrosive condition long before stored equipment is damaged.

Further research is in process to develop indicators of various sensitivities by changing the type and thickness of the metal films.

Radar System for Tracking and Forecasting Local Weather Disturbances.—A compact, easily-operated ground weather radar system, specifically designed to provide quick, accurate local weather conditions at distances up to 150 miles has been developed by the Radio Corporation of America, according to Arthur L. Malcarney, Executive Vice President, RCA Defense Electronic Products.

"This new ground weather surveillance radar, based on RCA's AVQ-10 airborne weather radar, will prove most valuable in predicting and tracking weather disturbances," Mr. Malcarney said. "It will have particular application in airport weather surveillance, television and radio station weather information transmission, construction projects and off-shore drilling operations.

"Easy to operate, easy to install and easy to maintain, the radar was designed and is being manufactured by our Defense Electronic Products West Coast Electronic Products Department in Los Angeles.

"The RCA Ground Weather Radar makes it possible to view and track storm fronts, thunderheads, squalls, heavy cloud formations and other forms of precipitation and weather disturbances.

"The compact system consists of antenna, receiver-transmitter, indicator, accessory unit and a control group. The antenna and receiver-transmitter are housed in a specially-built radome, which can be mounted on existing buildings or towers. Other units are housed in a control console.

"In operating the radar under storm conditions, the indicator scope first shows a pictorial representation of the storm. Then as the disturbance approaches, it is tracked and its direction and speed calculated. Nature and size of the storm, the extent of rainfall and other characteristics can be quickly determined from the information displayed on the viewing screen. After only a minimum of training an operator is able to identify and evaluate most storm types."

Pointing to the use of RCA's radar on construction projects as one example of the unit's versatility, Mr. Malcarney said that these operations are particularly vulnerable to storms because of insufficient knowledge of local weather conditions, and that losses could be reduced and completion schedules improved by dependence on information supplied by radar.

Other typical applications, he said, are agricultural areas, airports, dam sites, flight test and training activities, harbor operations and marine salvage, hurricane and tornado warnings, off-shore drilling, radio and TV stations, seaplane operations, private and university meteorological departments, and weather bureau services.

Ramjet Supersonic "Flight Tests." —Ramjet engines for advanced and future Air Force aircraft and missiles soon will be subjected to supersonic "test flights" with new equipment recently designed for the Air Research and Development Command. These "test flights" will be conducted at speeds up to 2500 mph, temperatures up to 650° F., and altitudes in excess of 90,000 ft, without actually leaving the ground. The new test equipment is to be installed in one of the engine test cells of ARDC's Arnold Engineering Development Center, Tullahoma, Tenn.

The device, called a "variable Mach number and angle-of-attack nozzle," is a modified and improved version of a similar nozzle which has been used in the past for testing smaller ramjets. The new nozzle was designed and built by General Dynamics Corporation's Electric Boat Div., Groton, Conn.

Improvements in the higher-performance nozzle are a higher temperature airflow, and its increased capability to simulate climbing and diving test conditions by changing the angle of the airflow entering the inlet of the jet engine. The new version will have a deflection of plus or minus fifteen degrees from the horizontal.

The nozzle has two flexible steel plates located horizontally between two flat stationary side walls. The airflow entering the test cell passes through the opening between the plates; and as the opening between the plates is decreased, the velocity of the airflow is increased. Speed can be increased from no flow to 2500 mph. in a matter of seconds.

At the same time the nozzle is simulating various speeds and maneuvers, the plant equipment generates and conditions the air supply to correspond to the type of air the engine would encounter if it were actually flying under these various conditions of climbing and diving.

The sealed test cell containing the new nozzle is designed to withstand explosions or fires, and even if an engine fails, hundreds of instruments

record information which can indicate the location of the failure.

In this way, the engine's early development can be done more safely, more rapidly and efficiently than in actual flight. This helps provide more reliable engines when actual free flights of the aircraft or missile are begun. It will assist in avoiding failures on initial test flights and missile firings which are expensive and often reveal no apparent cause for the failure.

Although primarily designed for ramjet testing, supersonic test conditions for turbojet engines can be simulated with the "variable Mach number and angle-of-attack nozzle."

The Twistor—A New Magnetic Memory Concept.—A new concept in memory devices has emerged from exploratory work by A. H. Bobeck at Bell Telephone Laboratories. This concept, which has been named the "Twistor," is expected to make possible memory systems which are simpler to fabricate and more economical to manufacture than existing systems. Such devices may have extensive applications in computers and electronic switching systems where rapid-access, high capacity memories are necessary.

The "Twistor" concept opens the way for the construction of magnetic memory arrays by merely interweaving horizontal copper wires and vertical magnetic wires, much as window screen is woven. Such a device would be similar in appearance to a ferrite core array, but without the cores, and would operate in much the same manner as a core array.

This new concept gets its name "Twistor" from a characteristic of wire made of magnetic material. Torsion applied to such a wire shifts the preferred direction of magnetization from a longitudinal to a helical path. The coincidence of a circular and a longitudinal magnetic field can then be used to insert information into this wire in the form of a polarized helical magnetization, and the magnetic wire itself can be used as a sensing means.

Application of torsion to the magnetic wire in a final device may be unnecessary, as the helical path for the preferred direction of magnetization can perhaps be "frozen" into the wire during processing.

In practice, the circular magnetic field is provided by a current pulse through the magnetic wire, and the longitudinal field by a current pulse through the copper wire which is perpendicular to the magnetic wire. Thus, storing a bit requires two coincident current pulses. One pulse by itself is insufficient to store a bit. Readout is accomplished by overdriving the longitudinal field in the reverse direction. The readout signal is sensed across the magnetic wire. Because the lines of magnetic flux along the helical path wrap the magnetic conductor many times, a favorable increase in the output signal is obtained.

Investigations are now under way to determine optimum size and composition for the magnetic wires. It appears that a conductor plated with magnetic material may have some advantages. Diameters as small as one-thousandth of an inch appear to be feasible. At least 10 bits per inch may be stored on such a wire without adverse interaction.

In conventional magnetic core memory devices, conductors must be threaded through the cores to make up a suitable matrix. When a ferrite sheet is employed, either a threading or a plating operation is necessary to

suitably locate the conductors. However, with the "Twistor," the ferrite material is completely eliminated and no threading or plating is necessary. Speed of operation and output of the "Twistor" are comparable to ferrite memory systems.

Present indications are that the drive circuits for a "Twistor" array can be readily transistorized. Thus, a memory system using the "Twistor" concept will retain all of the advantages of ferrite core or sheet systems, and will be much simpler and more economical to fabricate.

Gravity Measurements in the Open Sea.—The first successful surface measurements of gravity in the open sea were made on November 22 by J. Lamar Worzel of the Lamont Geological Observatory, the US-IGY Committee and Lamont have announced. The measurements were made as part of a program to chart gravity variations throughout the world during the International Geophysical Year.

A newly-developed sea gravimeter was used by Worzel to make the unprecedented surface measurements. The instrument was mounted on a gyro-stabilized platform installed aboard the USS *Compass Island*. Previously, gravity values for ocean areas had to be measured in submarines submerged to quiet depths.

The difficulty of obtaining and fitting submarines for this purpose reduced to a minimum gravity data for the 80 per cent of the earth's surface that is covered by water. The new sea gravimeter, developed by Anton Graf of Munich, Germany, now makes it possible to take gravity measurements simply and expeditiously any place on the ocean surfaces and thus assures that data long needed but difficult or impossible to obtain will now be available.

Surface gravity measurements from the USS *Compass Island* were taken in nine hours, as compared with two days for the earlier submarine measurements. The data were reduced in just one-half day, as compared with two weeks needed to adjust and compute data from submarine measurements.

To date there have been only 4000 measurements of gravity at sea made throughout the world. Roughly half of these have been made by the Lamont Geological Observatory in the past 10 years, using pendulums carried in submerged submarines.

Gravity measurements are used to determine the shape of the earth, the true direction of the earth's center, and the density of the material of the top-most layers. When gravity and seismic measurements are used together the thickness of the earth's crust can also be determined. Gravity determination of the true vertical and the shape of the earth are among the most important geodetic (mapping) tools we have. Once sufficient values of gravity are taken over a wide enough area of the oceans, maps and charts will be greatly improved.

The USS *Compass Island* left New York November 18 and returned November 22. The surface gravity measurements were begun 200 miles southeast of New York on November 22. They were continued on a track moving from the deep ocean basin across the continental slope and onto the continental shelf, ending off Long Island.

Data obtained with the new gravimeter on the tossing surface were checked against readings made in the same area aboard the submarine USS *Tusk* in 1947.

Packaged Circuits for Computer Design.—Miniaturized, low-cost packaged circuits for use in computer

design are now for sale by the Research and Advanced Development Division of Avco Manufacturing Corporation, of Lawrence, Mass.

Avco's "packaged transistorized logical building blocks" enable the computer engineer to use ready-made, completely compatible, logical elements in system design in the same manner that circuit engineers employ resistors and capacitors. With the building blocks, it is possible to jump from symbolic logic or Boolean algebra to finished equipment without an intermediate electronic development stage. Room-size computers may now be designed to desk dimensions with the building blocks. No air conditioning is needed, and each logical element occupies only 2½ cubic inches.

Eight basic building blocks, all measuring 2½ × 2 × ½ in., comprise the Avco series. Long life and trouble-free operation are incorporated in the circuits, since the electronic elements operate well within the performance limits of their transistors.

The eight types of building block are the bistable trigger element, monostable element, gated amplifier element, complementer amplifier element, "or" gate package, "and" gate package, "and-or" gate package, and emitter follower package. These basic units can be combined to produce virtually any logical system desired.

The elements operate over a temperature range of −50° C. to 85° C. At temperatures above 25° C., some derating of load current and switching speed is necessary.

This wide temperature range means that a building block computer could easily operate in tropic zones without extensive air conditioning. The only cooling requirement would be installation of a fan to circulate room-temperature air through the computer.

Two types of mounting sockets are available: solder lugs or dual-output taper pins. Rack mounting chassis can be supplied using either type socket for flush, recessed or enclosed mounting with interconnecting terminals located either front or back. Each rack is 3½ in. high, holds 25 building blocks, and fits a standard relay rack.

Avco also supplies a transistorized power unit capable of supplying up to 150 logical building blocks with six d-c. voltage outputs. The 7-in.-high unit fits a standard relay rack.

Brief descriptions of the eight basic Avco building blocks follow:

Bistable trigger element—Incorporating two stable output levels: 0 volts, which normally represents a binary 1; and −10 volts, which represents 0. The element has two separate inputs, 0 and 1, which may be joined externally to provide a "scale of two" circuit.

Gated amplifier element—Providing current gain and used when required after gating elements to give standard output pulses. Pulse retiming is possible by introducing a clock pulse to one input and the two-input "and" gate in front of this amplifier.

Complementer amplifier element—Providing a "not" or inhibit function, this element produces an output which is the complement of its input. Thus a 1 input produces a 0 output, and vice versa.

Monostable element—This element is essentially a one-shot pulse generator. Output voltage is at 0 in its quiescent condition, but when a negative pulse or step is provided at the input, a negative output pulse of fixed width is generated.

"And" gate package—Containing two 3-input "and" gates and two 2-input "and" gates. The gate will operate up to a maximum pulse repetition rate of one megacycle.

"Or" gate package—Containing two "or" gates, each having up to five outputs.

"And-Or" gate package—Combining the "and-or" function, this multi-use element saves components and is more compact. It will also perform the two functions separately. It contains four separate "and-or" gates.

Emitter follower package—Containing four complete, independent amplifiers, this element is used in conjunction with "and-or" operation. It is also useful in increasing driver capabilities of other building block units.

Device Reads Handwritten Numerals.—An experimental device the size of a portable typewriter that can read hand-written numerals or identify numerals as they are being written, has been invented by Tom L. Dimond of Bell Telephone Laboratories' System Engineering department. The machine was demonstrated at the Eastern Joint Computer Conference in Washington, in December. With some modifications the equipment could be used to read handwritten letters, Mr. Dimond said.

The machine may eventually become a valuable addition to telephone offices. It could be important in any situation where it is necessary to write and identify large quantities of numerals.

For example, each long distance ticket used in the Bell System contains 20 to 30 handwritten characters. Approximately two billion of these tickets are processed each year. A technique for picking up the information on these tickets with a machine rather than the human eye would be a tremendous aid in preparing telephone bills.

The new machine recognizes numbers as they are being written and indicates the numeral by lighting up the correct digit on a numbered panel. The writing is done with a metal stylus on a specially-prepared writing surface. Two dots, one above the other, are used as reference points. Seven sensitized lines extend radially from these two dots. Numerals are recognized by the machine, depending on which lines are crossed.

To clear the device for the next number, the writer touches the stylus to a special plate which causes the previous number to be "erased."

To recognize previously written numerals, such as those on a long distance ticket, it is necessary to write with a pencil containing a conductive lead. The ticket is then inserted in the machine into a special slot under a plate that has seven sensitized lines. The machine then uses the same principle for recognizing numbers already written as it does for the ones written on the machine with the stylus. It determines which sensitized lines have been crossed and indicates the proper number by illuminating a numeral on the lucite panel. This information could be transferred to an accounting machine, computer or other data-processing device.

Several experimental models have been built at Bell Laboratories. Each is completely self-contained.

The device is operated from flashlight batteries and requires no outside power source. Its small size is made possible by the use of transistors.